Spatially Distributed Snowmelt Modeling with the Utah Energy Balance Snowmelt Model

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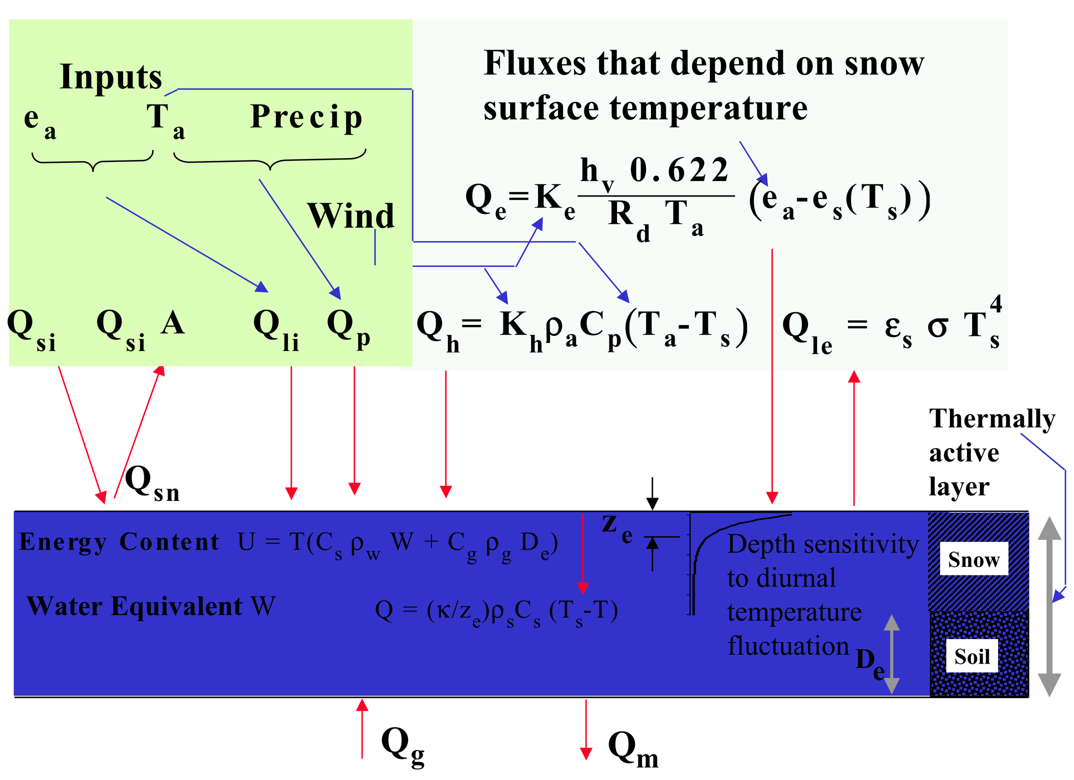
Abstract

This paper describes some improvements that have been made to the Utah Energy Balance (UEB) Snowmelt model in the way that snow surface temperature is modeled. The Utah Energy Balance snowmelt model is a single layer snowmelt model designed to be parsimonious for spatially distributed grid applications. In the model snowmelt is driven by surface energy fluxes that depend strongly on surface temperature. Recognizing that surface temperature is different from an average or representative single layer snow temperature the model has to date used an equilibrium gradient approach to parameterize surface temperature. Comparisons against measurements of internal snow temperature revealed that this scheme led to deficiencies in the modeling of snowpack internal energy. This paper describes new components added to the model to address these deficiencies. We have changed the parameterization of surface temperature from an equilibrium gradient approach to a modified force restore approach. We have also added a simplified representation of the advance of a refreezing front during periods of heat loss following melt. These parameterizations retain the simple one layer property of the model, important for parsimony, but improve the comparisons between measured and modeled internal energy, snow surface temperature, melt outflow and snow water equivalent. This model has been applied to the simulation of snowpack on a spatially distributed grid over the Green Lakes Valley watershed in Colorado as part of an effort to understand the spatial distribution of snow and parameterize the subgrid variability of snow processes for application with larger model elements.

UEB single layer point snowmelt model (Tarboton et al, 1995; Tarboton and Luce, 1996)



Mass Balance Equation

$$\frac{dW}{dt} = P_r + P_s - M_r - E$$

Energy Balance Equation

$$\frac{dU}{dt} = \underbrace{Q_{sn} + Q_{li} - Q_{le} + Q_p + Q_h + Q_e}_{Q_{forcing}(T_s)} - Q_m + Q_g$$

Surface temperature (T_s) by equilibrium gradient approach was solved through:

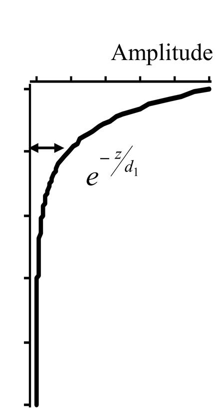
$$Q_{cs} = K_s(T_s - \overline{T}) = Q_{forcing}(T_s)$$

Theory of heat conduction into snow (Luce, 2000; Luce and Tarboton 2001b)

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} \qquad k = \frac{\lambda}{C\rho}$$

$$T(z,t) = \langle T\rangle + Ae^{-\frac{z}{d_1}}\sin\left(\omega_1 t - \frac{z}{d_1}\right)$$

$$\omega_1 = 2\pi/24\text{hr} \qquad d_1 = \sqrt{\frac{2k}{\omega_1}}$$



$$Q_{cs} = -\lambda\frac{dT}{dz}\Big|_{z=0} = \frac{\lambda}{d_1\omega_1}\frac{dT_s}{dt} + \frac{\lambda}{d_1}(T_s - \langle T\rangle)$$

Force restore approach

$$Q_{cs} = \frac{\lambda}{d_1\omega_1}\left(\frac{T_s - T_{s_{lag1}}}{\Delta t}\right) + \frac{\lambda}{rd_1}\left(T_s - \overline{T}\right)$$

(Finite difference approximation to time derivates Substitute depth average snow temperature, $\overline{T}$, for mean <T> in sinusoidal solution)

Modified force restore approach

$$Q_{cs} = \frac{\lambda}{d_1\omega_1}\left(\frac{T_s - T_{s_{lag1}}}{\Delta t}\right) + \frac{\lambda}{rd_1}\left(T_s - \overline{T}_s\right) + \frac{\lambda}{d_{lf}}\left(\overline{T}_s - \overline{T}_{24}\right)$$

where $d_{lf} = \sqrt{2k/\omega_{lf}}$ and the low frequency ω_{lf} is calibrated. (Finite difference approximation to time derivative. Substitute 24 hours average surface temperature for mean <T> in sinusoidal solution. Include term for superimposed gradient with lower frequency driven by difference between 24 hour averages of surface ($\overline{T}_s$) and snow ($\overline{T}_{24}$) temperatures.)

Equilibrium gradient approach

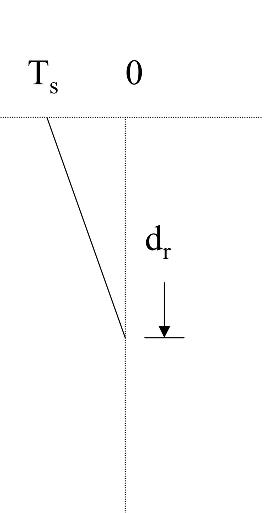
$$Q_{cs} = \frac{\lambda}{rd_1}\left(T_s - \overline{T}\right)$$

r ≈ 1 calibrated parameter

(Ignore time derivative, substitute depth average snow temperature, $\overline{T}$, for mean <T> in sinusoidal solution)

Theory of refreezing front propagation

The presence of liquid water in snow inhibits the depression of surface temperature and enhances heat loss. In periods where the forcing $Q_{forcing}(T_s)$ has switched to negative, in the presence of liquid water (U>0) we model the penetration of a refreezing front.



Assumptions:

- Dependence of forcing on T_s is linearized
  $Q_{forcing}(T_s) = a - bT_s$
- Linear temperature gradient in layer above freezing front
  $Q(T_s) = \lambda\frac{T_s}{d_r}$
- All energy loss goes to latent heat of refreezing (heat capacity of refreezing snow neglected)
- Melt water density based on liquid holding capacity with depth of wet layer from quantity of liquid water present.
- New surface melt ($Q_{forcing}(T_s)$ >0) resets d_r to 0.

With these:

$$Q(T_s) = Q_{forcing}(T_s) \Rightarrow T_s = \frac{a}{\lambda/d_r + b}$$

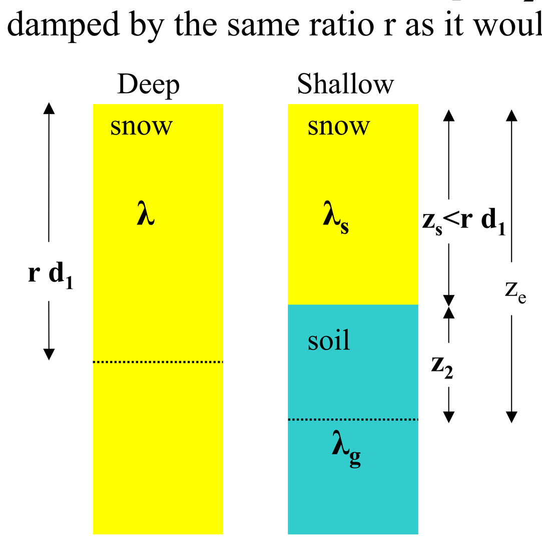
$$\frac{d\,d_r}{dt} = -\frac{Q(T_s)}{\rho_m L_f} = \frac{a - b\dfrac{a}{\lambda/d_r + b}}{\rho_m L_f}$$

$$\Rightarrow$$

$$d_r = \frac{-\lambda + \sqrt{\lambda^2 + 2b\left(\lambda d_{r1} + \frac{b}{2}d_{r1}^2 - \frac{a\lambda\Delta t}{\rho_m L_f}\right)}}{b}$$

Theory of adjustments of λ for shallow snow

Where snow is shallow the implied depth (rd_1) over which the gradient acts may extend into the ground. In these cases we use an effective thermal conductivity λ_e as the harmonic mean to the depth z_2 where amplitude is damped by the same ratio r as it would be for deep snow.



$$k_g = \frac{\lambda_g}{\rho_g C_g}$$

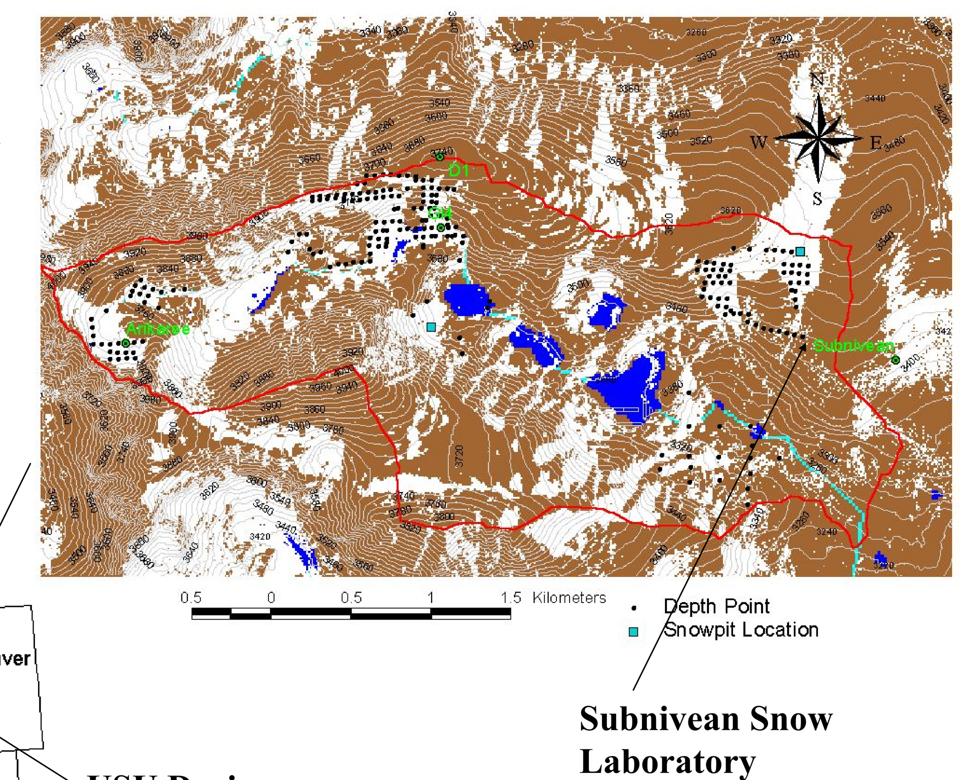
$$z_2 = \sqrt{\frac{2k_g}{\omega_1}}\left(r - \frac{z_s}{d_1}\right)$$

$$z_e = z_s + z_2$$

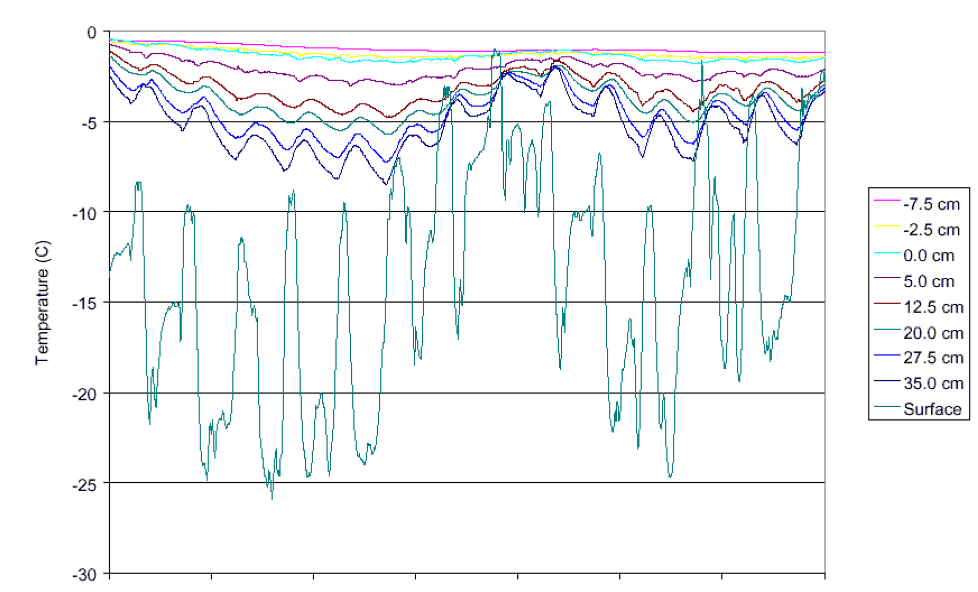
$$\frac{1}{\lambda_e} = \frac{\dfrac{z_s}{\lambda_s} + \dfrac{z_2}{\lambda_g}}{z_e}$$

Study site and model results

Data from Utah State University Drainage Farm (USU DF), UT, Central Sierra Snow Laboratory (CSSL), CA, and Subnivean Snow Laboratory in Green Lakes Valley (GLV) watershed, CO were used in the model calibrating and testing.

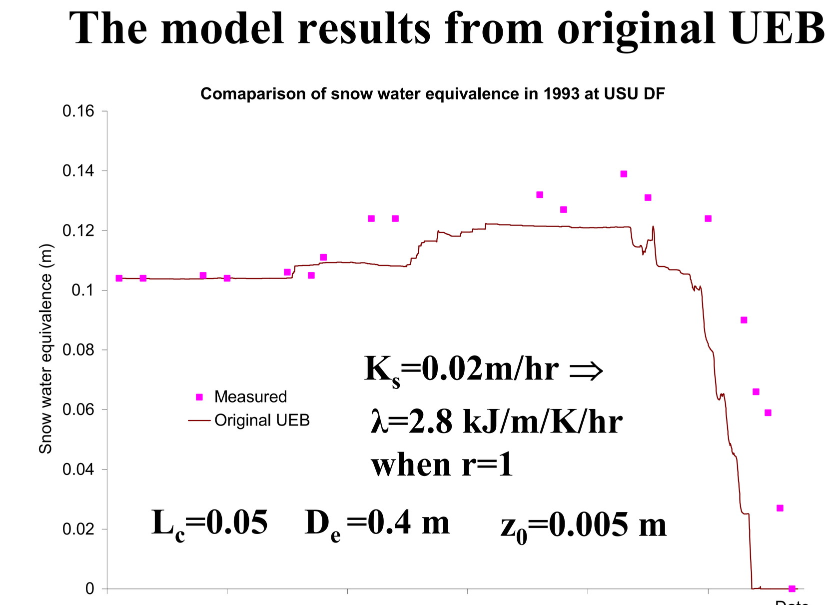


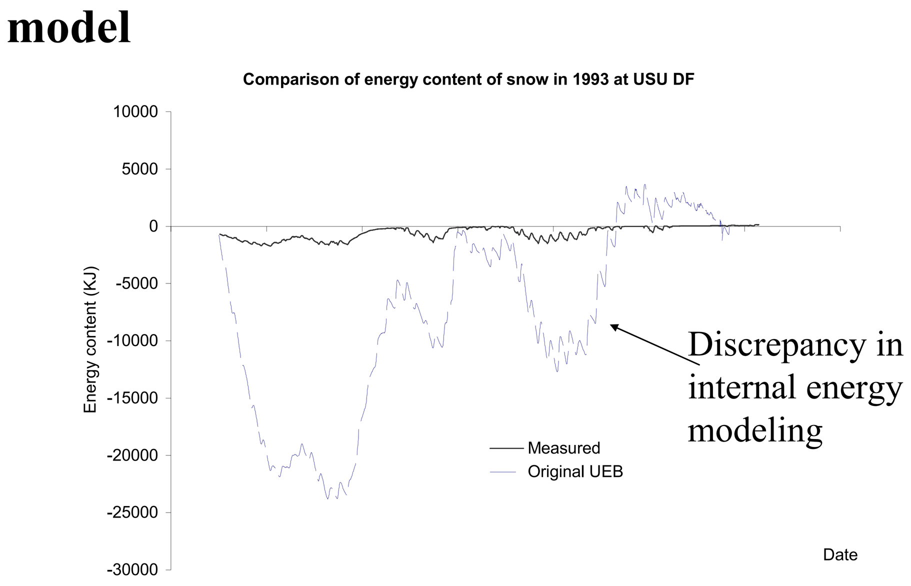
Observed temperature profile of snowpack at USU DF was used to estimate the internal energy of the snowpack.



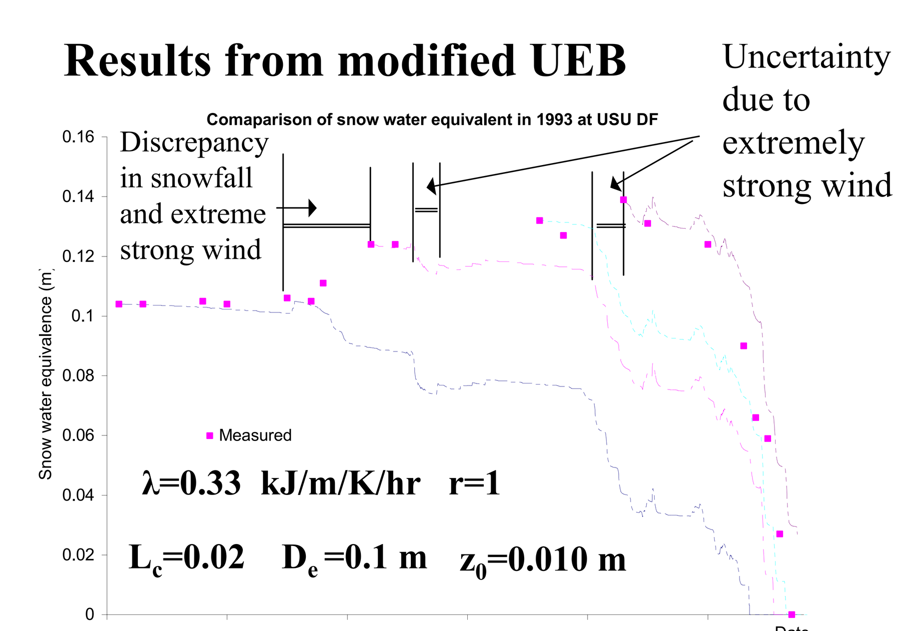
Model results

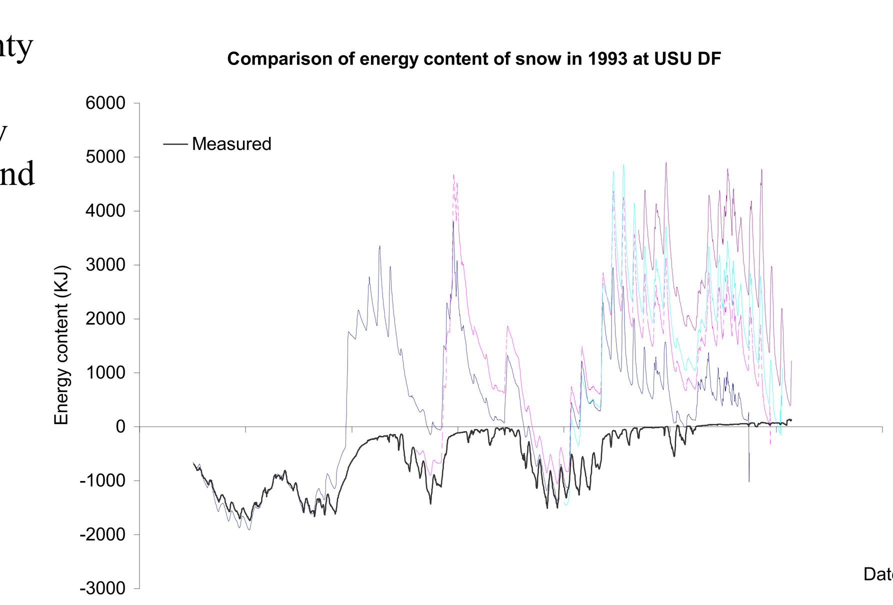
The model results from original UEB model



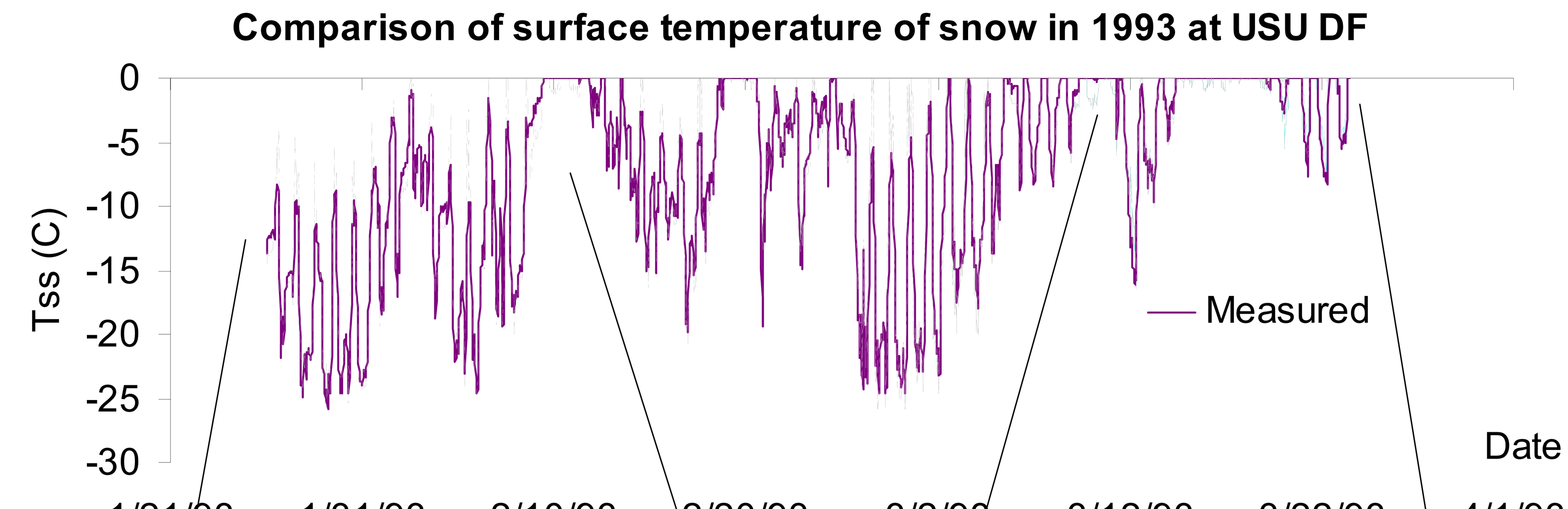


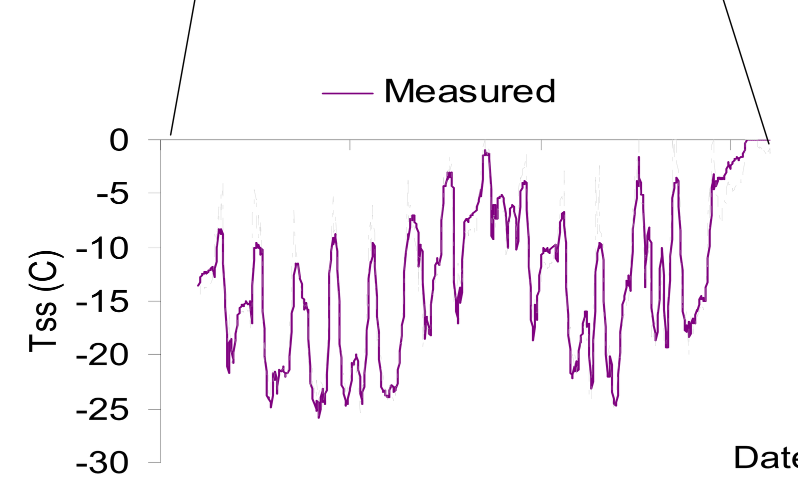
Results from modified UEB

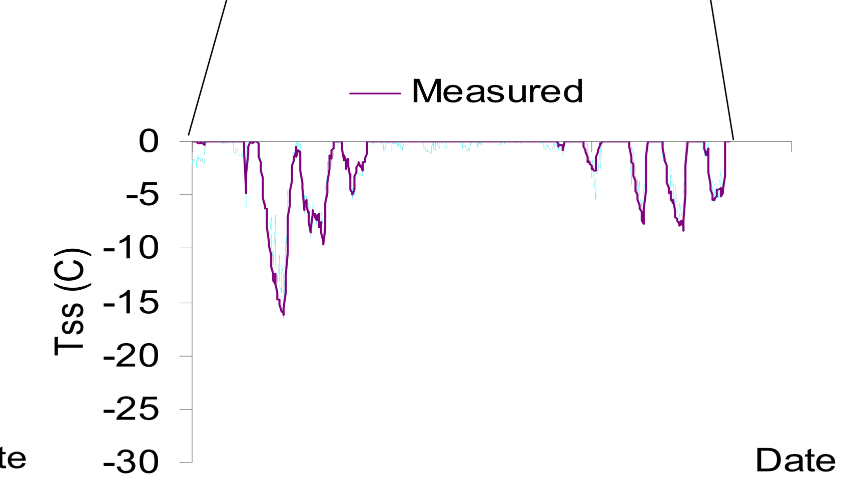


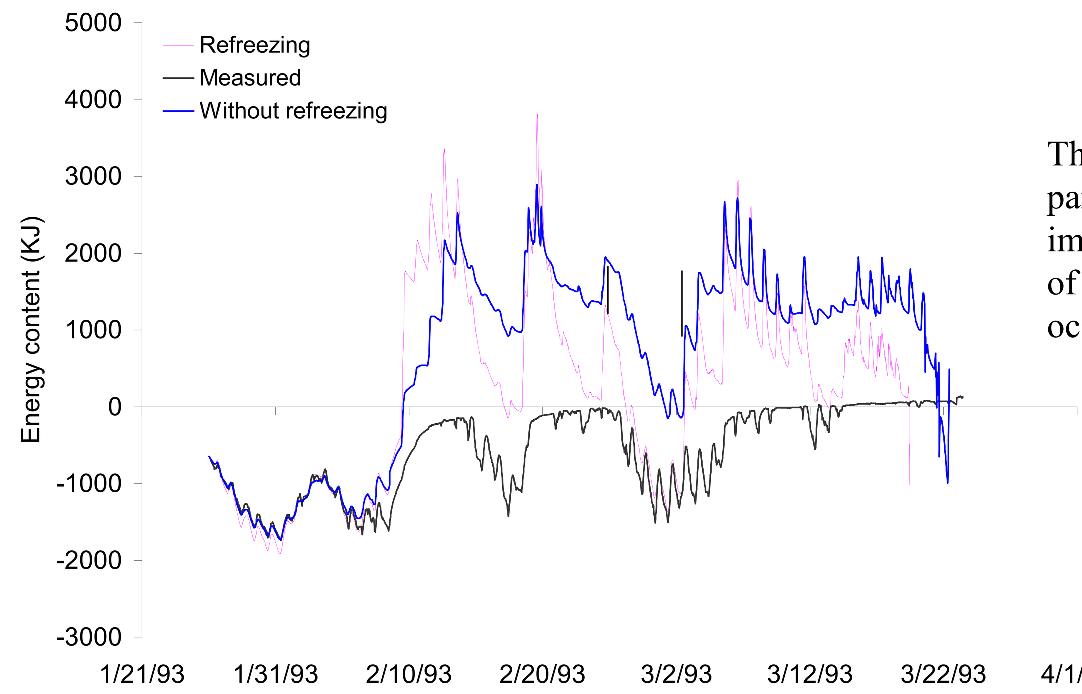


Comparison of surface temperature of snow in 1993 at USU DF



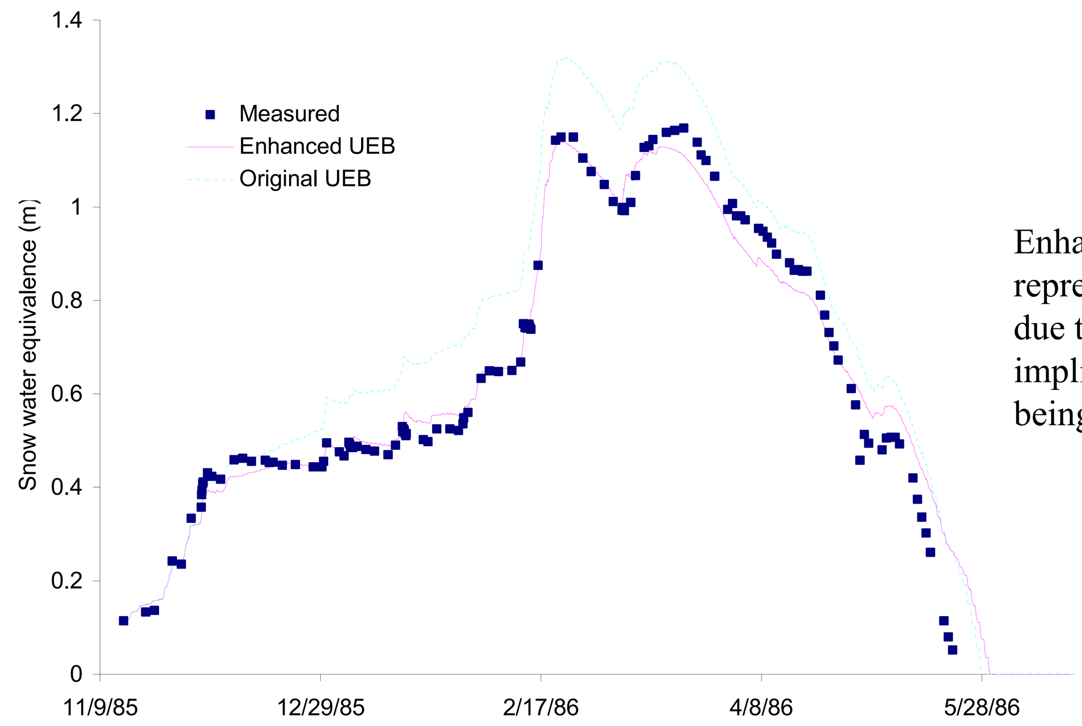






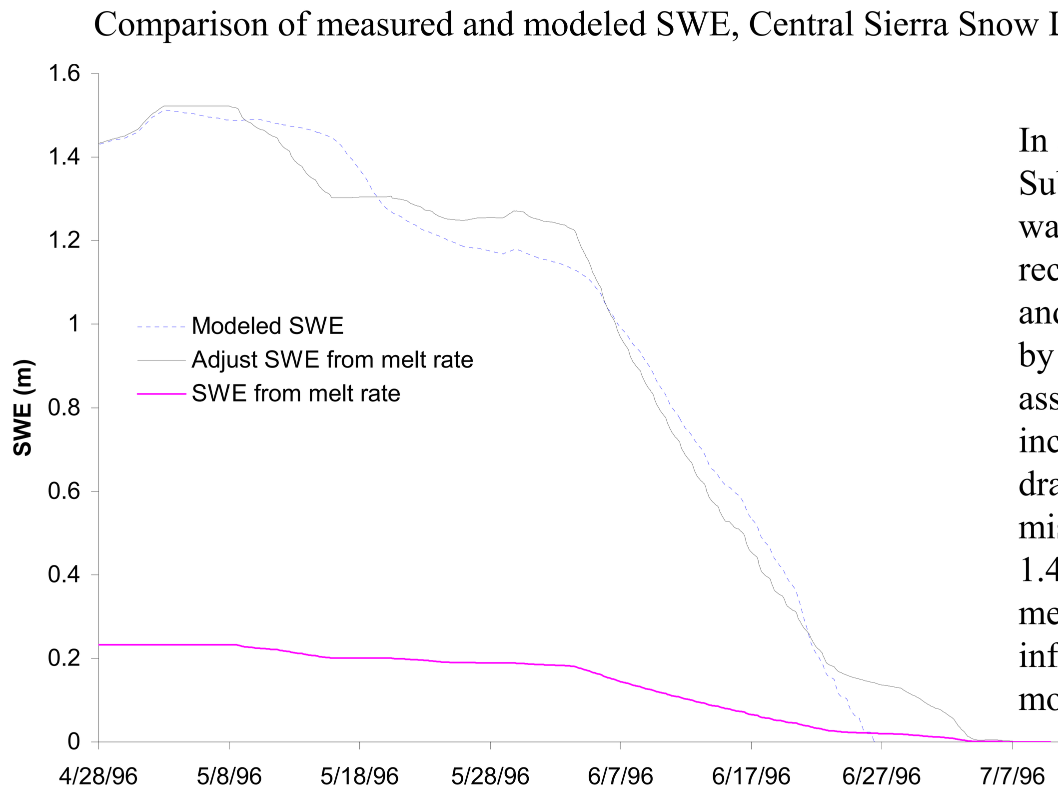
The refreezing parameterization improved the modeling of heat loss following occurrence of some melt.

Comparison of measured and modeled snow energy content, USU Drainage Farm.



Enhanced model better represents early season losses due to energy content and implied snow temperatures being close to melting.

Comparison of measured and modeled SWE, Central Sierra Snow Laboratory

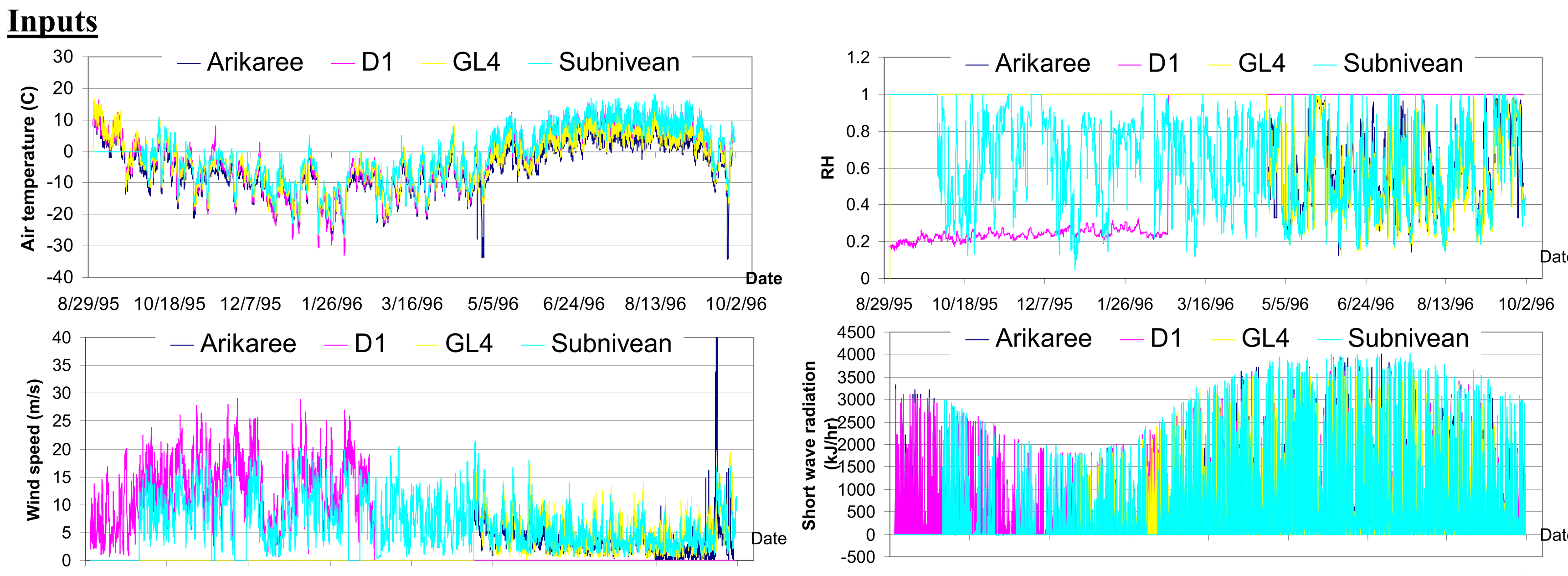


In applying at the Niwot Ridge Subnivean snow laboratory there was a large discrepancy between recorded initial SWE (1.43 m) and total melt outflow recorded by the lysimeter (0.23 m). We assumed that the lysimeter was incorrect due to preferential drainage in the snowpack being missed, so adjusted (scale up by 1.43/0.23) the lysimeter measurements to derive an inferred SWE to compare to the model.

Comparison of inferred and modeled SWE, Niwot Ridge

Spatial distributed snowmelt modeling

Inputs



Spatial measurements

The measurement in Green Lakers Valley watershed includes:

1) The snow depth measurement at 269 points
2) Climatic forcing data (air temperature, relative humidity, wind speed, and incidental shortwave radiation.) at four metrological stations.
3) Snow covered area images at four date. (high resolution air borne images)

Method

- Apply model on distributed grid over watershed to learn about spatial variability
- Model accounts for topographic effects on snowmelt processes (radiation and temperature)
- To account for spatial variability of snow accumulation due to drifting and sliding we use the drift factor approach.

Drift factor approach

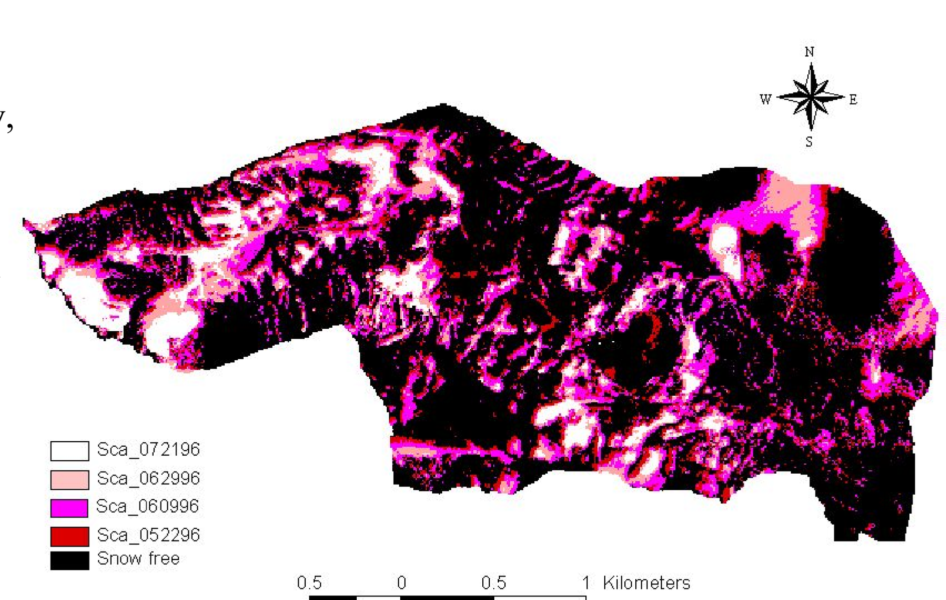
The precipitation was separated into snowfall or rainfall through:

$$f_{snow} = \begin{cases} 1.0 & \text{when } T_a<T_s \\ \dfrac{T_r - T_a}{T_r - T_s} & \text{when } T_s<=T_a<=T_r \\ 0.0 & \text{when } T_a>T_r \end{cases}$$

where f_{snow} is the fraction of the precipitation as snow. T_r(=3 °C) is the air temperature above which all precipitation is assumed to fall as rain, and T_s(=-1 °C) is the air temperature below which all precipitation is assumed to fall as snow. Snowfall is adjusted for wind induced drifting, using the drift factor ϕ for each grid cell, and is given as:

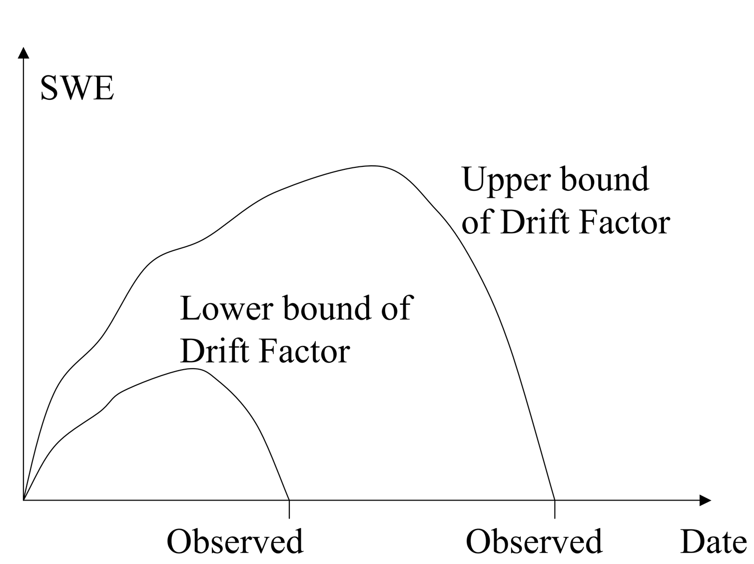
$$p_{snow} = f_{snow} \cdot \phi \cdot p$$

where P is the measured precipitation (m). The total precipitation at each cell is the sum of P_{snow} and precipitation as rainfall.



Green Lakes Valley Snow Cover observations from aerial photography

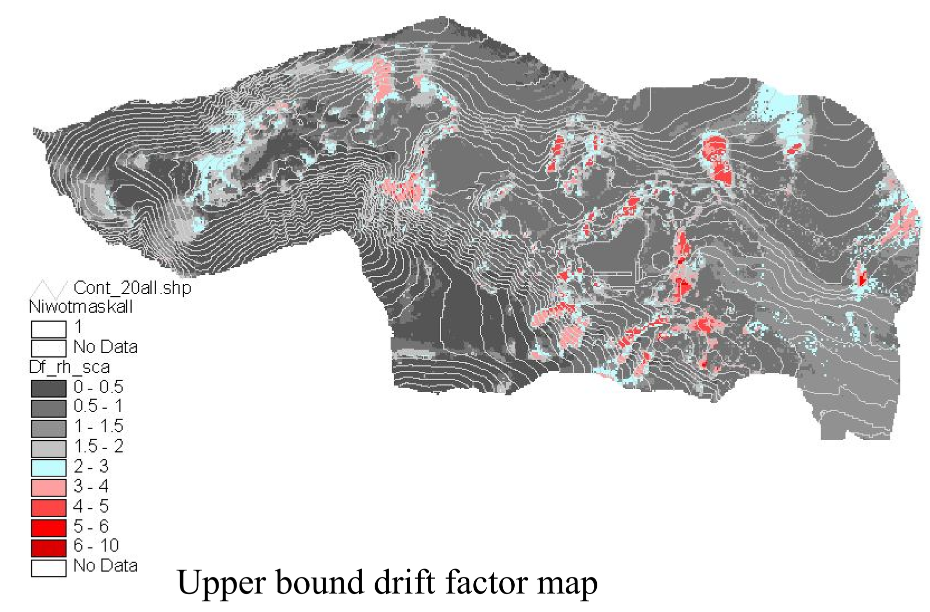
Here bounds on drift factor are estimated from when the snow disappears as recorded in aerial photography. The lower bound on drift factor is that value that has snow disappearing on the last day snow cover was observed. The upper bound on drift factor is that value that has snow disappearing on the first day snow was not observed.



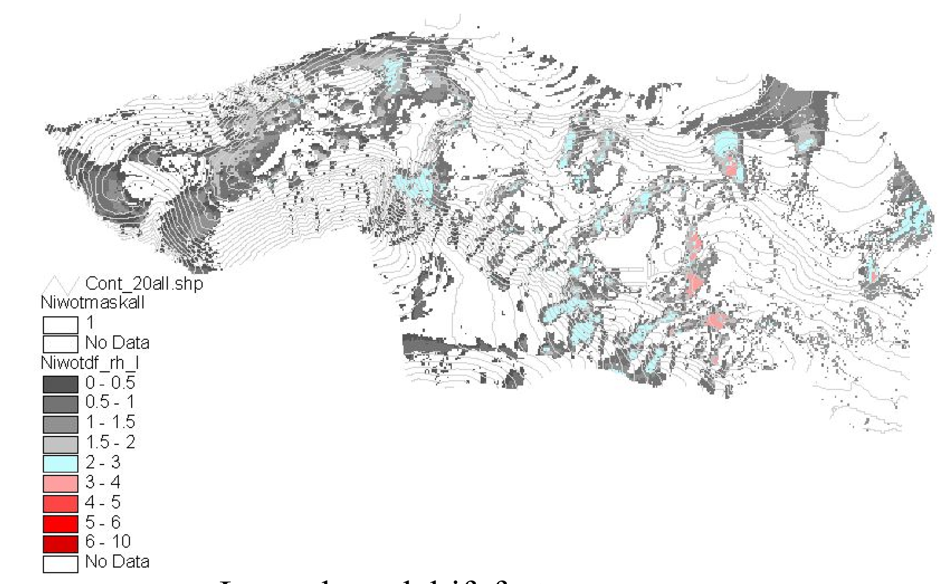
Green Lakes Valley

Results

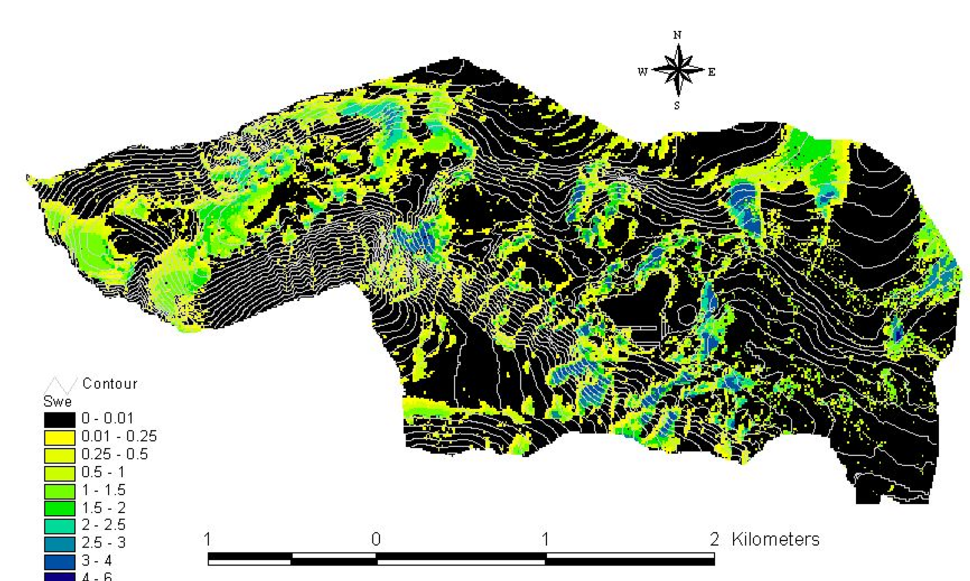
The model was run with both lower and upper bound drift factors to bracket the possible range.



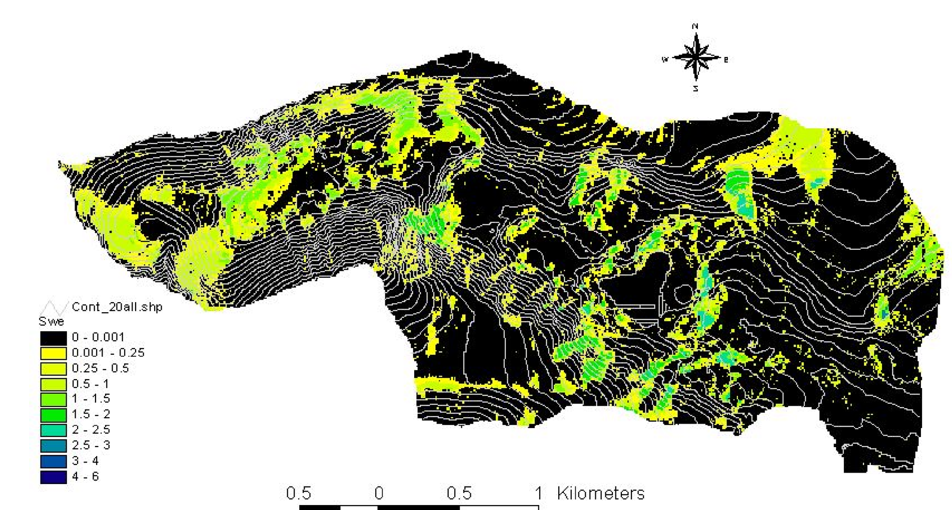
Upper bound drift factor map



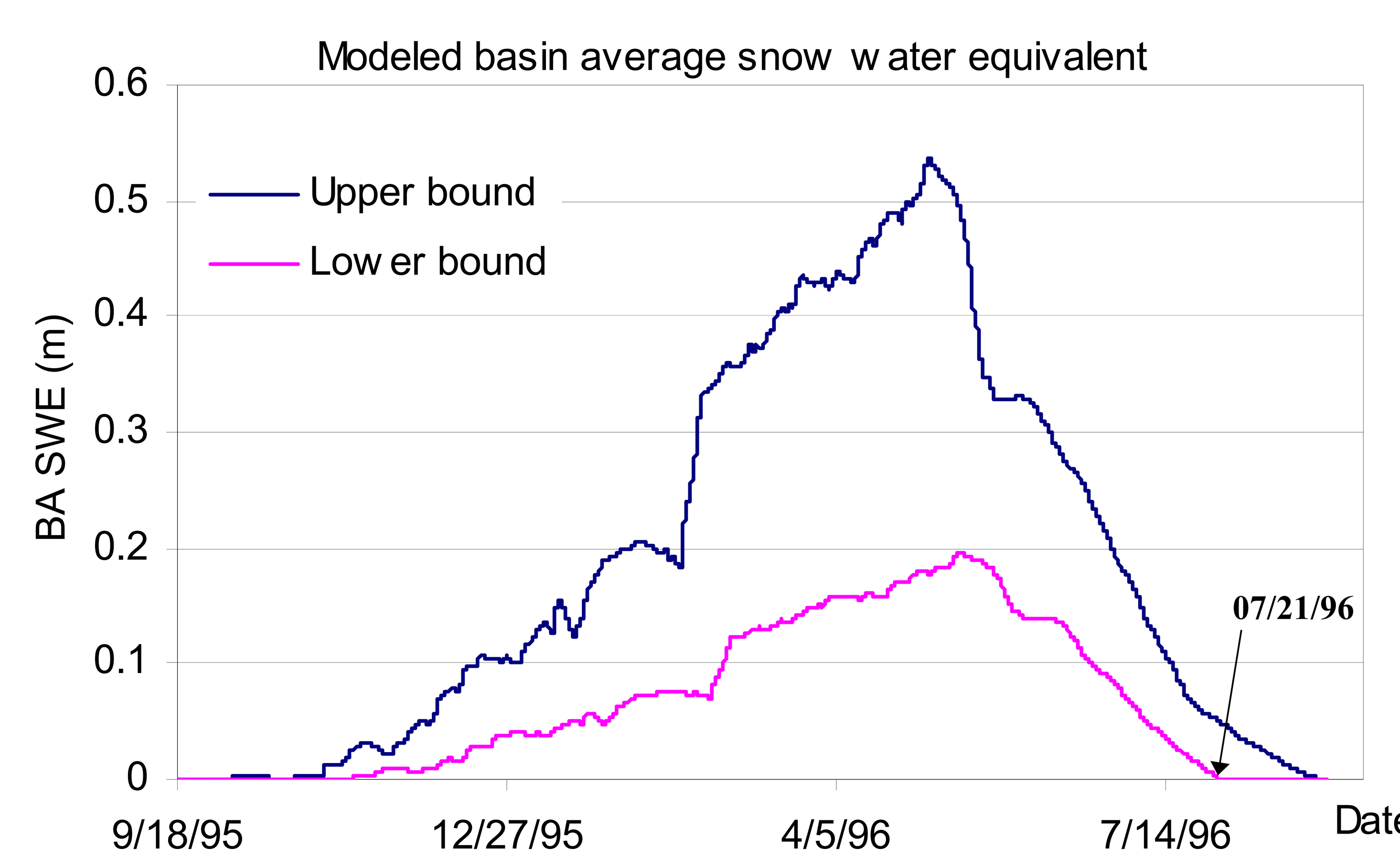
Lower bound drift factor map



Modeled SWE at May 22, 1996 with upper bound drift factor



Modeled SWE at May 22, 96 with lower bound drift factor



Comparison of basin average snow water equivalent with input of upper bound and lower bound of drift factor

Conclusions:

1. Modified force restore surface temperature of snow was introduced. Results show that this results in better modeling of internal energy of snowpack.
2. Refreezing front propagation parameterization was introduced. Results shows better modeling of internal energy during the post melt time period.

Ongoing work

1. Exploring relationship between drift factor and topography
2. Examining distribution of snow and related depletion curves (Luce et al. 1999, Luce, 2000, Luce and Tarboton, 2001a)
3. Exploring relationships between depletion curves as subgrid parameterization and topography.

References (see http://www.engineering.usu.edu/dtarb/)

Luce, C. and D. Tarboton, (2001a), "Scaling Up Snowpack Accumulation and Melt Models," Submitted to Water Resources Research.

Luce, C. H. and D. G. Tarboton, (2001b),"A Modified Force-Restore Approach to Modeling Snow-Surface Heat Fluxes," Proceedings of the 69th Annual Western Snow Conference, Sun Valley, Idaho.

Luce, C. H., (2000), "Scale Influences on the Representation of Snowpack Processes," PhD Thesis, Civil and Environmental Engineering, Utah State University.

Luce, C. H., D. G. Tarboton and K. R. Cooley, (1999), "Subgrid Parameterization Of Snow Distribution For An Energy And Mass Balance Snow Cover Model," Hydrological Processes, 13: 1921-1933, special issue from International Conference on Snow Hydrology, Brownsville, Vermont, 6-9 October, 1998.

Tarboton, D. G. and C. H. Luce, (1996), "Utah Energy Balance Snow Accumulation and Melt Model (UEB)," Computer model technical description and users guide, Utah Water Research Laboratory and USDA Forest Service Intermountain Research Station.

Tarboton, D. G., T. G. Chowdhury and T. H. Jackson, (1995),"A Spatially Distributed Energy Balance Snowmelt Model," in Biogeochemistry of Seasonally Snow-Covered Catchments, ed. K. A. Tonnessen et al., Proceedings of a Boulder Symposium, July 3-14, IAHS Publ. no. 228, p.141-155.

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