Terrain Analysis Using Digital Elevation Models in Hydrology[1]

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This paper describes methods that use digital elevation models (DEMs) in hydrology, implemented as an ArcGIS toolbar using Visual Basic and the ESRI object library. I describe generalized channel network delineation to objectively estimate drainage density and by using terrain curvature accommodate spatially variable drainage density. The multiple flow direction field determined from a DEM also serves as a basis for routing overland and topographically driven subsurface flow useful in water quality, erosion and terrain stability modeling. New DEM derived quantities, such as downslope influence, upslope dependence, decayed accumulation, downslope accumulation and transport limited accumulation are illustrated.

Introduction

Terrain analysis based on digital elevation models is being increasingly used in hydrology (e.g. Wilson and Gallant, 2000). This is driven by the availability of digital elevation data, nationally from the USGS (2003) and worldwide including space based data available from the NASA TOPSAR shuttle mission. This is also driven by the increasing computer power available in personal computers with the capability to rapidly download and process digital elevation model (DEM) data and use topographic attributes extracted from DEMs in hydrologic models.

This paper describes generalized methods for channel network delineation that implement objective procedures for drainage density estimation. This paper also describes new DEM derived quantities useful in water quality, erosion and terrain stability modeling based on the D∞ multiple flow direction model (Tarboton, 1997).

Channel Network Delineation

Hydrologic processes are fundamentally different on hillslopes and in channels. In channels flow is concentrated. The drainage area, A, (e.g. in m^2) contributing to each point in a channel may be quantified. On hillslopes flow is dispersed. The "area" draining to a point is zero because the width of a flow path to a point disappears. On hillslopes flow and drainage area need to be characterized per unit width (e.g. $m^3/s/m = m^2/s$ for flow). The specific catchment area, *a*, is defined as the upslope drainage area per unit contour width, *b*, ($a = A/b$) (Moore et al., 1991) and has units of length (e.g. $m^2/m = m$). Figure 1 illustrates these concepts.

The differences between processes on hillslopes and in channels make it important to properly map the physical extent of channels in a watershed. Model elements in hydrologic and water quality models are sometimes delineated based on area draining directly to a channel segment with hillslope or overland flow length a parameter used to quantify for example hydrologic response time or erosion and sediment delivery (e.g. hillslope length in the USLE methodology Wischmeyer and Smith, 1978). The correct scale associated with the terrain needs to be identified, so that model input parameters are estimated correctly.

[1] Presented at 23rd ESRI International Users Conference, July 7-11, 2003, San Diego, California.

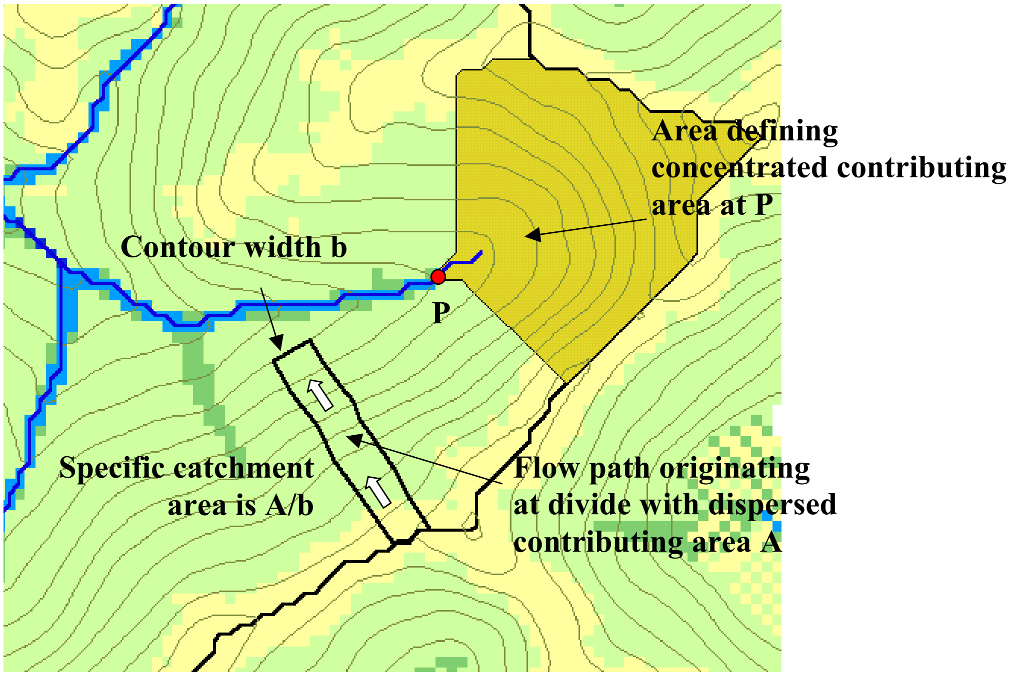


Figure 1. Definitions of concentrated and dispersed contributing area and specific catchment area.

Tarboton and Ames (2001) present ways to objectively delineate drainage networks from digital elevation models that respect this distinction between hillslopes and channels. A grid of local curvature is used as a weight grid in a drainage area accumulation function with a support threshold to delineate the channel network and watersheds. The support threshold is chosen objectively using the constant drop property for Strahler streams.

Mapping channel networks from digital elevation models follows the now well rehearsed procedure (e.g. Wilson and Gallant, 2000; Tarboton and Ames, 2001) of filling pits, computing flow direction and then computing the contributing area draining to each grid cell. The earliest method (O'Callaghan and Mark, 1984) for delineating drainage networks used a support area threshold applied to the grid of drainage areas. Channels and channel start points are mapped as those grid cells where the support area threshold is exceeded. This procedure has been widely used and is implemented in Arc Hydro (Maidment, 2002). A significant question with this method is what support area threshold to use. Figure 2 illustrates this issue where drainage networks with two different support area thresholds are depicted.

Tarboton et al. (1991) suggested methods based on the relationship between slope and contributing area, and the constant stream drop property to objectively decide upon a support area threshold. This procedure brings objectivity to the procedure but is still limited because the drainage density of the network extracted is still spatially uniform.

Tarboton and Ames (2001) suggested identification of local curvature as a method to account for spatially variable drainage density. Upwards curved grid cells have been used by others to depict channel networks from digital elevation data (Band, 1986; Wilson and Gallant, 2000). However patterns of locally upwards curved grid cells are disconnected and not readily amenable to network analysis. Upwards curved grid cells can be formed into a network by using them as a weighting field in a weighted drainage area computation. A threshold in this weighted drainage area is used to map and delineate channels. The weighted support area threshold used to map channels is chosen objectively using the constant drop property of channel networks (Broscoe, 1959). The smallest weighted support area threshold that produces a channel network where the mean stream drop in first order streams is not statistically different from the mean stream drop in higher order streams, is selected. Stream drop is defined as the difference in elevation between the beginning and end of Strahler streams. The Strahler (1952) stream ordering system designates source streams as first order; the confluence of two (or more) first order streams is the beginning of a second order stream; the confluence of two (or more) second order streams produces a third order stream and so on. When a stream of a given order receives a tributary of lower order, its order does not change. A Strahler stream is defined as an entire set of sequential stream segments with the same order.

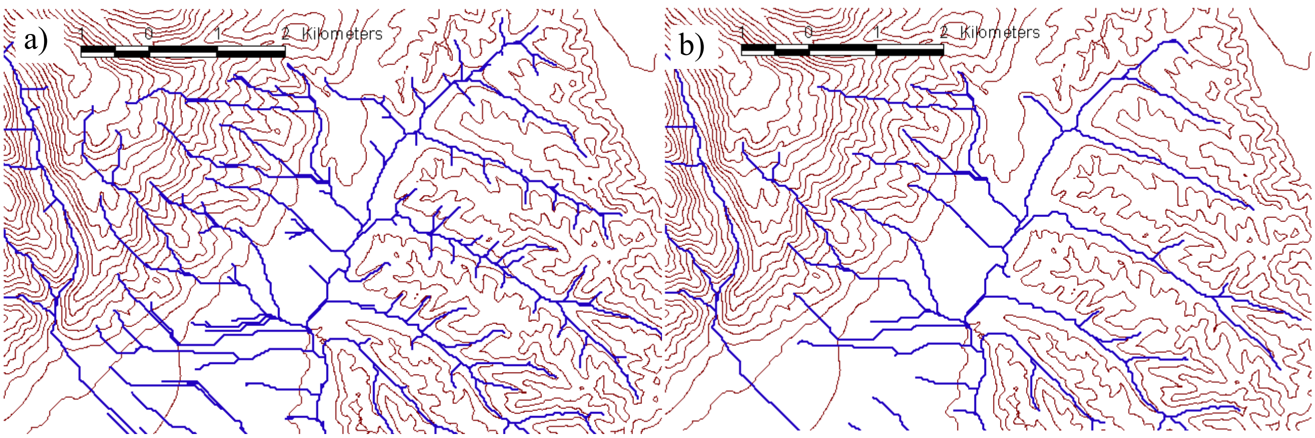


Figure 2. Mawheraiti River, New Zealand. 40 m contours, DEM with 30 m grid size based on Contours supplied by Land Information, New Zealand. Drainage networks delineated with a) 100 gridcell, b) 300 grid cell threshold.

The constant drop property is an empirical geomorphological attribute of properly graded drainage networks, that has a physical basis in terms of geomorphological laws governing drainage network evolution (Tarboton et al., 1992). By using the smallest weighted support area that produces networks consistent with this property we are extracting the highest resolution drainage network statistically consistent with geomorphological laws. A smaller weighted support area threshold would result in drainage networks with first order stream drops inconsistent with the rest of the drainage network. When such a network is mapped one observes that stream seem to extend up what appear to be smooth hillslopes. A weighted support area larger than required for consistency with the constant drop law results in a coarse drainage network that omits drainage paths from what contour examination would indicate to be valley forms where concentrated flow occurs.

Upwards curved grid cells are identified using the algorithm due to Peuker and Douglas (1975) reported by Band (1986), illustrated in figure 3. This algorithm flags the pixel of highest elevation from each possible square of four adjacent grid cells. After one sweep of the matrix the unflagged grid cells represent drainage courses. Wilson and Galant (2000) suggest some alternative measures of curvature based on second derivatives of the surface. These could be used with a similar procedure.

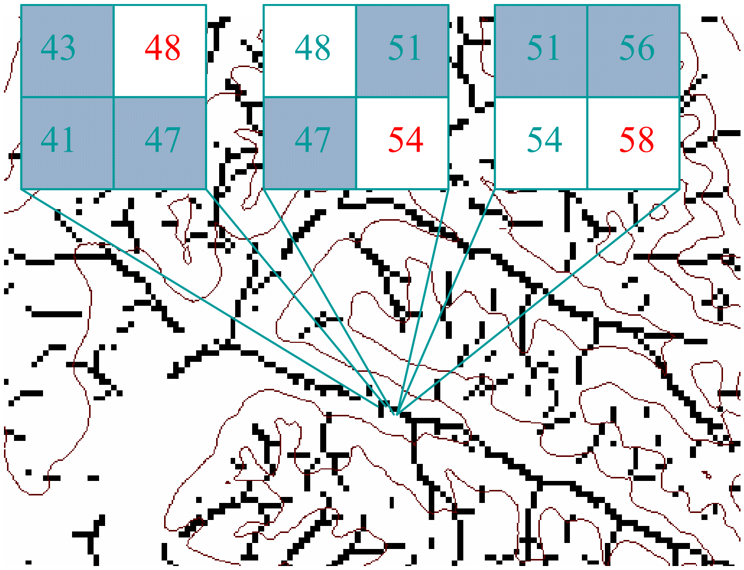


Figure 3. Peuker and Douglas (1975) method for identification of valley grid cells through a single sweep flagging (white) the highest grid cell in each set of four. The remaining unflagged (black) grid cells indicate valleys.

Figure 4 presents a constant drop analysis where a t test has been used to evaluate the difference in mean stream drop between first and higher order streams. The 95% confidence level for t tests is essentially 2. Based on this test the curvature weighted support area threshold of 20 grid cells is selected for the Mawheraiti study area, because it is the smallest support area threshold where the absolute value of the t statistic is less than 2 indicating that the mean drop between first order and higher order streams is not significantly different. The resulting channel network is shown in figure 5. Notice the adaptation of the procedure to the contour crenulations with lower drainage density relative to figure 2 in the flat areas, but comparable drainage density in the hilly areas. The Peuker Douglas procedure identified more upwards curved grid cells in the hilly areas where the contours are more crenulated than in the broad valley. The weighted accumulation of these resulted in higher drainage density in the hilly areas and fewer spurious looking drainage paths in the broad valleys.

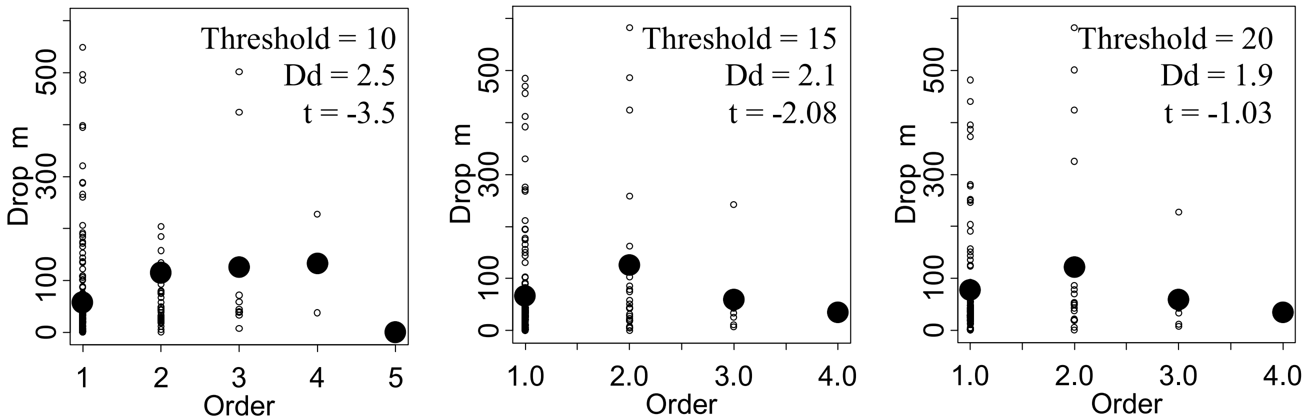


Figure 4. Stream drop test for Mawheraiti River. For each upward curved support area threshold the stream drop for each stream is plotted against Strahler stream order. The large circles indicate mean stream drop for each order The weighted support area threshold, drainage density (in km^{-1}) and t statistic for the difference in means between lowest order and all higher order streams is given.

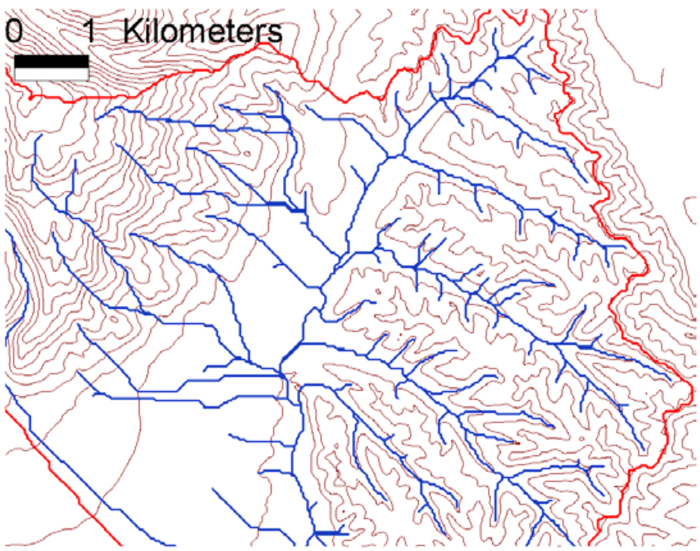


Figure 5. Mawheraiti River. 40 m contours. Drainage network delineated with upward curved weighted support area threshold of 20 grid cells.

Multiple flow direction accumulation functions

The procedures for channel network delineation are based on the D8 model for flow over a terrain surface represented by a grid DEM (O'Callaghan and Mark, 1984). In this model a single flow direction in the direction of steepest slope towards one of the eight (cardinal and diagonal) grid cells neighboring is used to represent the flow field. The D8 approach has disadvantages arising from the discretization of flow into only one of eight possible directions, separated by 45° (e.g. Fairfield and Leymarie, 1991; Quinn et al., 1991; Costa-Cabral and Burges, 1994). Tarboton (1997) introduced the D∞ multiple flow direction model (Figure 6) for the representation of flow within a DEM. Rather than representing flow in one of the eight possible directions from a grid cell to an adjacent or diagonal neighbor (D8) this procedure represents flow direction as a vector along the direction of the steepest downward slope on eight triangular facets centered at each grid cell. An infinite number or flow directions, represented as an angle between 0 and 2π are possible. Flow from a grid cell is shared between the two, downslope grid cells closest to the vector flow angle based on angle proportioning.

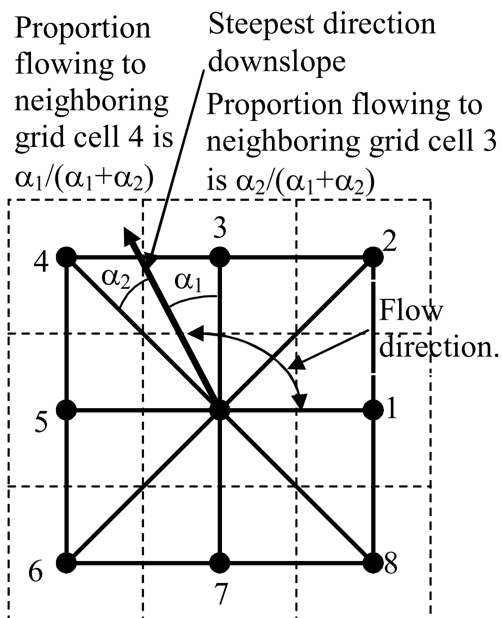


Figure 6. D∞ multiple flow direction model (Tarboton, 1997). Flow direction defined as steepest downwards slope on planar triangular facets on a block centered grid.

The D∞ multiple flow direction model is useful for the calculation of specific catchment area where flow is dispersed on a hillslope and can be readily extended to include weighted accumulation. Given a weighting field r(x) (which may for example represent excess rainfall, e.g. rainfall minus infiltration) the accumulation of r at each point on the land surface may be evaluated at each point as

$$A[r(x)] = \int_{CA} r(x)dx$$

A[.] is a functional operator that takes as input a spatial field r(x), and the topographic flow direction field (not denoted) and produces a field A(x) representing the accumulation of r(x) up to each point x. Integration is over the contributing area CA. Numerically each cell on a square grid is assigned a set of (one or more) flow proportions, representing the proportion of flow in each direction. There is a single value of r(i, j) associated with each grid cell. The accumulation value A(i,j) for each cell is evaluated

$$A[r(x)] = A(i,j) = r(i,j)\Delta^2 + \sum_{k \text{ contributing neighbors}} p_k A(i_k, j_k)$$

p_k is the proportion of flow from neighbor k contributing to the grid cell (i,j). The condition $\sum p_k = 1$ is required to ensure conservation. Directions must be assigned to ensure that there are no loops. Δ represents the grid cell size.

Figure 7 compares the specific catchment area calculated from D∞ with that obtained from D8. Qualitatively the contributing area calculated using the D∞ approach is smoother and avoids bias associated with the grid directions. Tarboton (1997) presents quantitative tests based on hypothetical cases where the specific catchment area is known demonstrating the effectiveness of the D∞ approach.

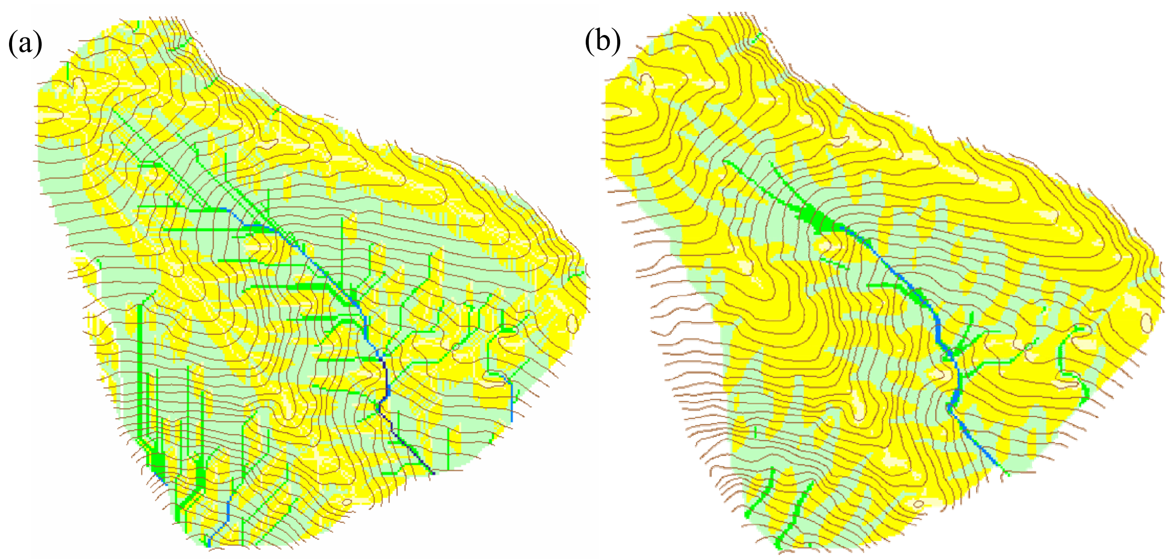


Figure 7. Contributing area computed using (a) the D8 flow direction model and (b) the D∞ flow direction model.

The D∞ multiple flow direction model may also be generalized based upon the concept of weighted flow accumulation to evaluate quantities useful in water quality analysis and land management. Figure 8 illustrates the downslope influence function. The Downslope Influence function (or influence zone) of a set y within the domain is defined as

$$I(x|y) = A[i(x|y)]$$

where A[] is the weighted accumulation operator evaluated using the D∞ Contributing Area function. I(x|y) says what the contribution from the set of points y is at each point x in the map. i(x|y) is an indicator (1,0) function on the set y and I is evaluated using the weighted contributing area function only on points in the set y. The downslope influence function is useful to track where sediment or contaminant moves.

The upslope dependence function is illustrated in figure 9. This function quantifies the amount a point x contributes to the point or zone y. It is the inverse of the downslope influence function

$$D(x|y) = I(y|x)$$

The upslope dependence function is useful to track, for example, where sediment or contaminant to a site may come from.

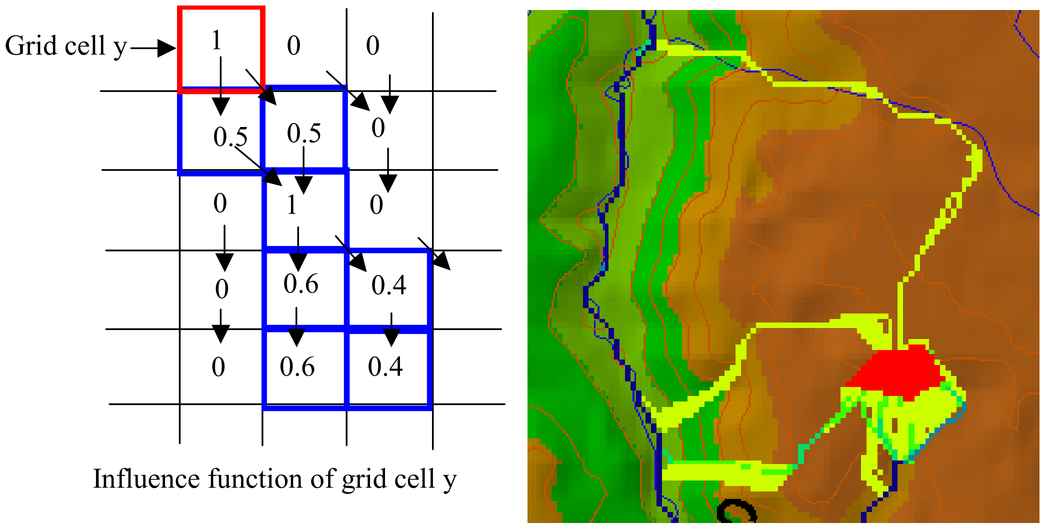


Figure 8. Downslope Influence Function.

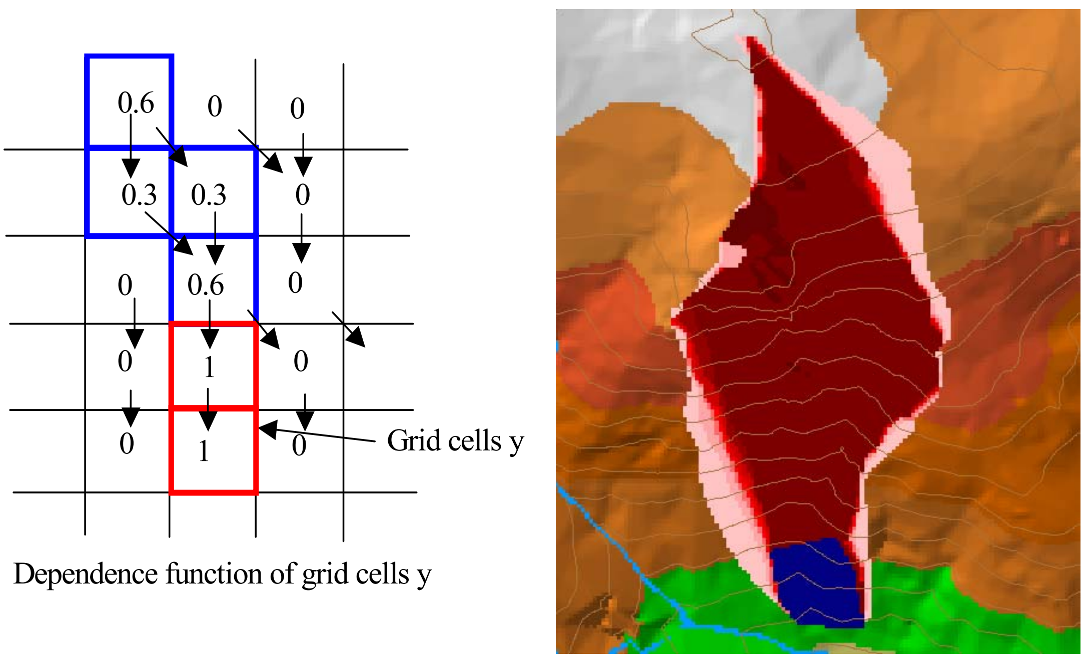


Figure 9. Upslope Dependence Function.

The decaying accumulation function is illustrated in Figure 10. This is designed to accumulate the loading of a substance that moves with flow but is subject to attenuation or degradation processes such as die off (in the case of fecal coliforms) or volatilization (in the case of chemical spills). This function is useful for tracking contaminant or compound subject to decay or attenuation as it moves with the flow. For example concentrations of gasoline spilled in a transportation accident will be reduced due to volatilization as the gasoline is transported downslope. This function is also useful in the case of industrial spills.

A decaying accumulation operator DA[.] is defined that takes as input a mass loading field m(x) expressed at each grid location as m(i, j) that is assumed to move with the flow field but is subject to first order decay in moving from cell to cell. The output is the accumulated mass at each location DA(x). The accumulation of m at each grid cell can be numerically evaluated

$$DA[m(x)] = DA(i, j) = m(i, j)\Delta^2 + \sum_{k \text{ contributing neighbors}} p_k d(i_k, j_k) DA(i_k, j_k)$$

Here d(x) = d(i ,j) is a decay multiplier giving the fractional (first order) reduction in mass in moving from grid cell x to the next downslope cell. If travel (or residence) times t(x) associated with flow between cells are available d(x) may be evaluated as $\exp(-\lambda t(x))$ where λ is a first order decay parameter.

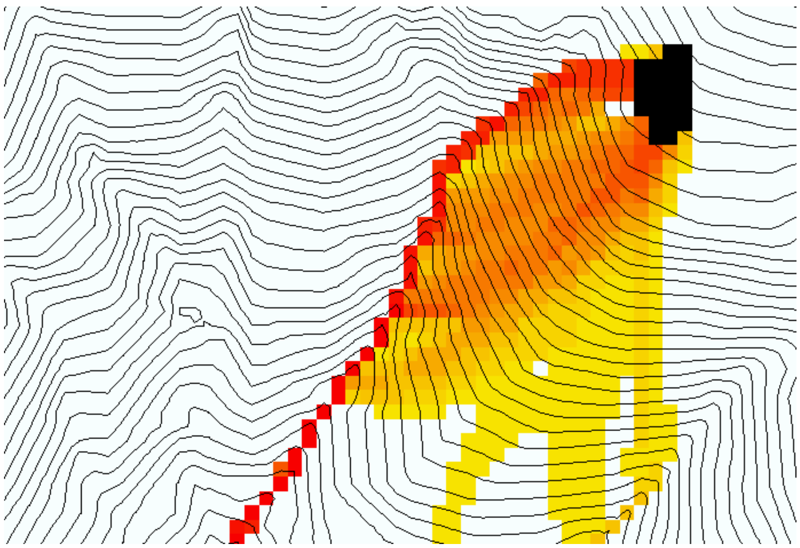


Figure 10. Decaying accumulation function output.

The concentration limited accumulation function is illustrated in Figure 11. This function applies to the situation where an unlimited supply of a substance (such as from an animal feeding operation) is loaded into flow at a concentration or solubility threshold C_{sol} over a set of points y. For example the concentration of fecal coliforms leaving an animal feeding operation site may be fixed at a concentration (e.g. 10^6/100 ml). Water flowing over the site of a gasoline spill leaves the site with a fixed concentration of BTEX compounds based on their relative abundance in the fuel spilled. Downstream concentrations are reduced due to dilution and volatilization. The flow specific discharge (volume per unit width) is taken to be a weighted accumulation of the flow input loading w(x), that represents, for example excess rainfall (rainfall – infiltration).

$$Q(x)=A[w(x)]$$

The set of points y, delineating the area of the substance supply are mapped using the (0,1) indicator field i(x;y). The concentration, C(x), and loading, L(x), of the substance at these locations is then given by

$$C(x) = C_{sol}$$

$$L(x) = C_{sol}\, Q(x)$$

This loading is then taken to move with the flow with loading and concentration downstream calculated accumulation subject to decay if appropriate.

$$L(x) = L(i, j) = \sum_{k \text{ contributing neighbors}} p_k d(i_k, j_k) L(i_k, j_k)$$

$$C(x) = L(x)/Q(x)$$

Here d(x) = d(i ,j) is a decay multiplier giving the fractional (first order) reduction in mass in moving from grid cell x to the next downslope cell, similar to the decaying accumulation function. The concentration limited accumulation function is useful for a tracking a contaminant released or partitioned to flow at a fixed threshold concentration.

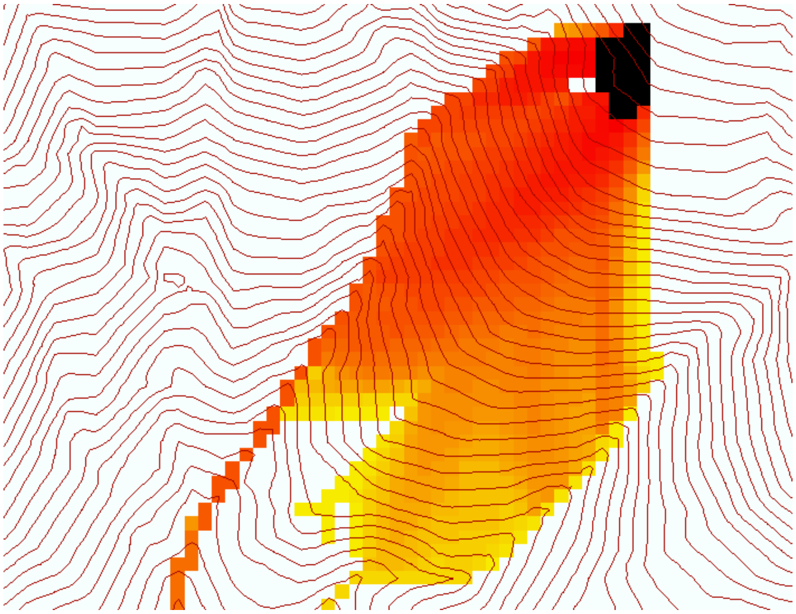


Figure 11. Concentration limited accumulation function output

The transport limited accumulation function is illustrated in figure 12. This function applies to the situation where there is a supply of substance (e.g. erosion) and capacity for transport of the substance (e.g. sediment transport capacity). This function accumulates the substance flux subject to the rule that the transport out of any grid cell is the minimum of the transport in to that grid cell and the transport capacity. There is then deposition in the amount of the difference.

$$T_{out} = \min(E + \sum T_{in}, T_{cap})$$

$$D = E + \sum T_{in} - T_{out}$$

Here E is the supply and T_{cap} the transport capacity. T_{out} at each grid cell becomes T_{in} for downslope grid cells and is reported as Transport limited accumulation. D is deposition at each grid cell. The function provides the option to evaluate concentration of a compound (contaminant) adhered to the transported substance. This is evaluated as follows

$$L_{in} = \sum T_{in} C_{in}$$

Where L_{in} is the total incoming compound loading and C_{in} and T_{in} refer to the Concentration and Transport entering from each upslope grid cell. If $T_{out} < \sum T_{in}$ then there is no erosion from the cell, so

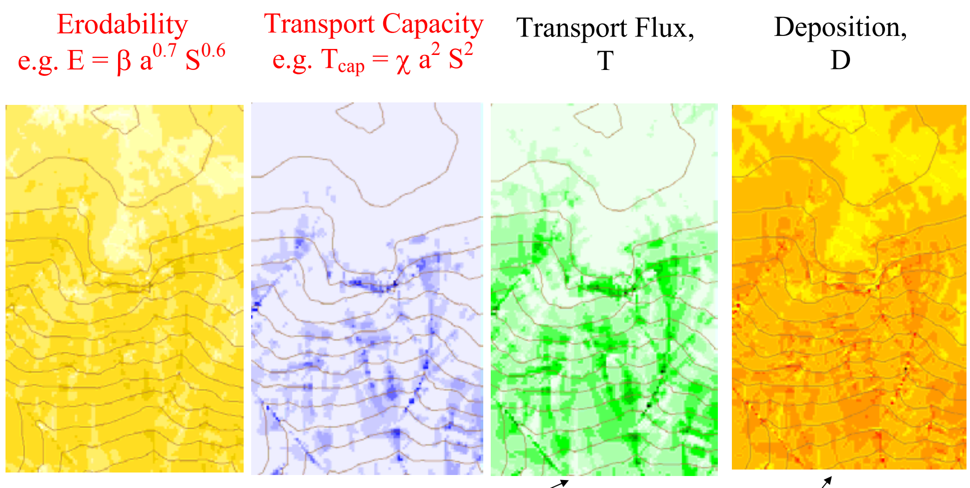
$$L_{out} = L_{in} \left(T_{out} / \sum T_{in} \right)$$

else

$$L_{out} = L_{in} + C_s \left(T_{out} - \sum T_{in} \right)$$

where C_s is the concentration supplied locally and the difference in the second term on the right represents the additional supply from the local grid cell. Then $C_{out} = L_{out} / T_{out}$

C_{out} at each grid cell comprises is the concentration grid output from this function. Transport limited accumulation is useful for modeling erosion and sediment delivery, including the spatial dependence of sediment delivery ratio and contaminant that adheres to sediment.



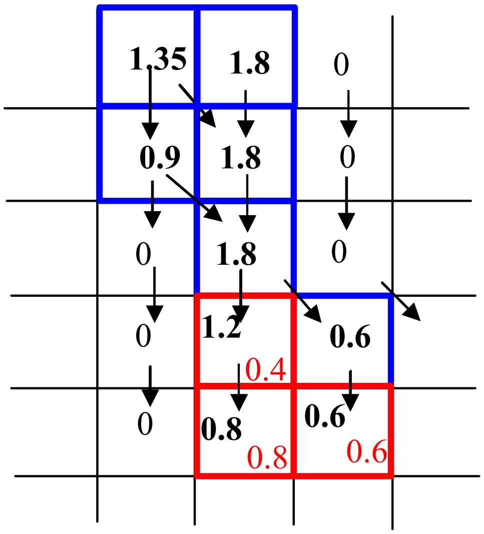
$$T_{out} = \min(E + \sum T_{in}, T_{cap}) \quad D = E + \sum T_{in} - T_{out}$$

Figure 12. Transport limited accumulation function.

The reverse accumulation function is designed to evaluate and map the hazard due to activities that may have an effect downslope. The example is land management activities that increase runoff. Runoff is sometimes a trigger for landslides or debris flows, so one is interested in measuring the amount of unstable terrain downslope from each location. The function takes as input a sensitivity weight grid, that could for example be a measure of terrain stability. The reverse accumulation provides a measure of the amount of unstable terrain downslope from each grid cell, as an indicator of the potential for danger due to activities that may have downslope impact even though there may be no potential for any local impact. The reverse accumulation function is illustrated in figure 13 and is similar to the evaluation of weighted contributing area, except that the accumulation is by propagating the weight loadings upslope along the reverse of the flow directions to accumulate the quantity of weight loading downslope from each grid cell. The function accumulates loading from the weight grid in excess of a weight threshold. The function also reports the maximum value of the weight loading downslope from each grid cell.

Implementation

The procedures presented have been programmed in C++ as a library of functions compiled into a component object model (COM) dynamic link library that is callable from other COM compliant systems such as Visual Basic and ESRI ArcGIS. The software accesses data in the ESRI grid data format directly using the GRIDIO application programmers interface that is part of ArcView. An ArcMap toolbar extension has been developed using Visual Basic to provide graphical user interface access to the functionality presented from within ArcMap. This toolbar is available from http://moose.cee.usu.edu/taudem/taudem.html.



Reverse accumulation of field weights indicated in red

Figure 13. Reverse Accumulation Function

Conclusions

This paper has described a method for the delineation of drainage networks based on the weighted accumulation of upwards curved grid cells. This method is adaptive to spatial variability in drainage density. The weighted support area threshold is chosen objectively using a t test to select the highest resolution drainage network with mean drop of first order streams not significantly different from the mean drop of higher order streams. In this way a drainage network consistent with geomorphology is delineated without the need to subjectively choose a support area threshold parameter. New functions based on the D∞ flow model useful in water quality, erosion and terrain stability modeling have been described.

Acknowledgements

I would like to thank Dan Ames, Brian Smith and John Cole for programming assistance. Bob Pack conceived and is thanked for support to develop the reverse accumulation function. This work was partly supported by the Idaho National Engineering and Environmental Laboratory and USGS State Water Research Center program.

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