A Sediment Transport Model for Incising Gullies On Steep Topography

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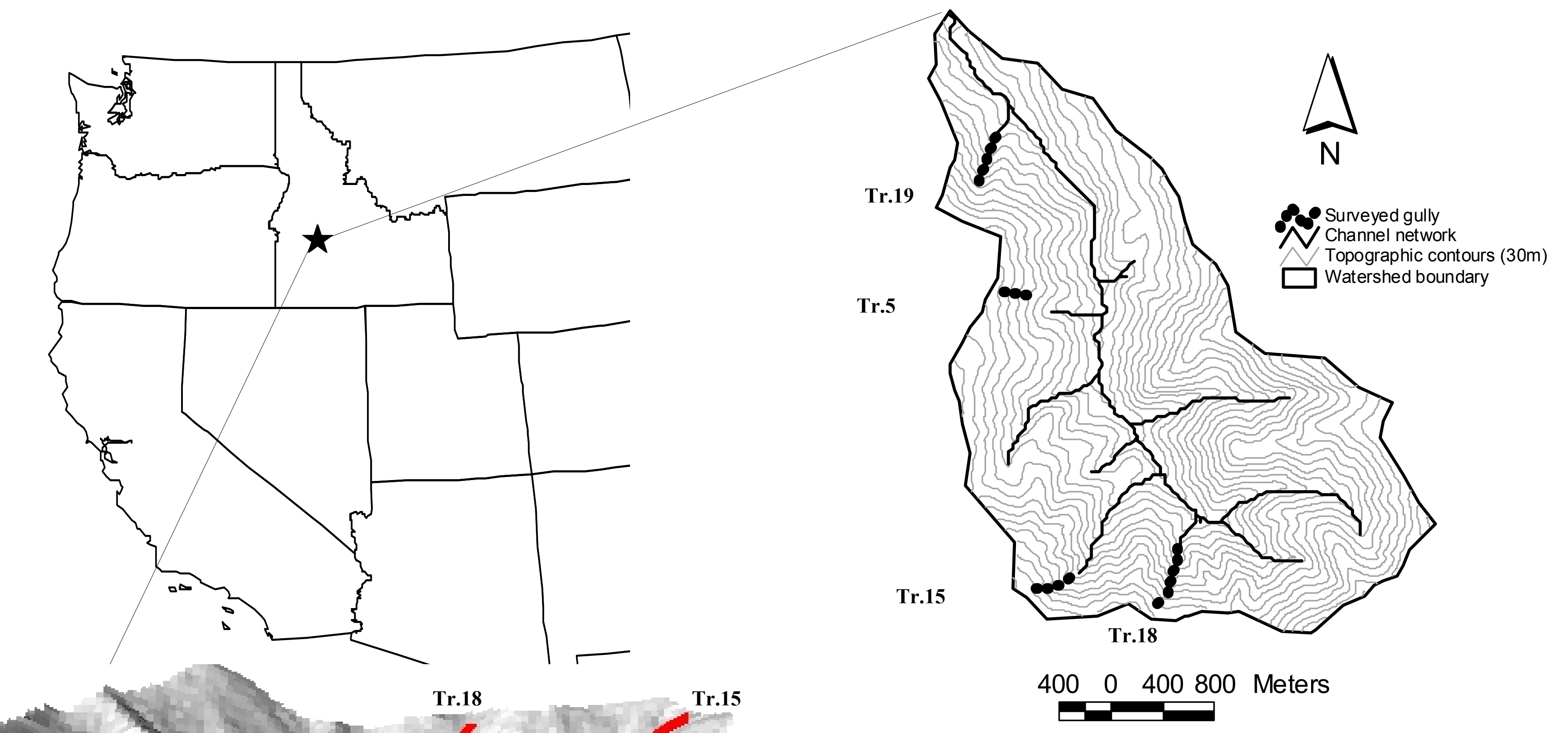
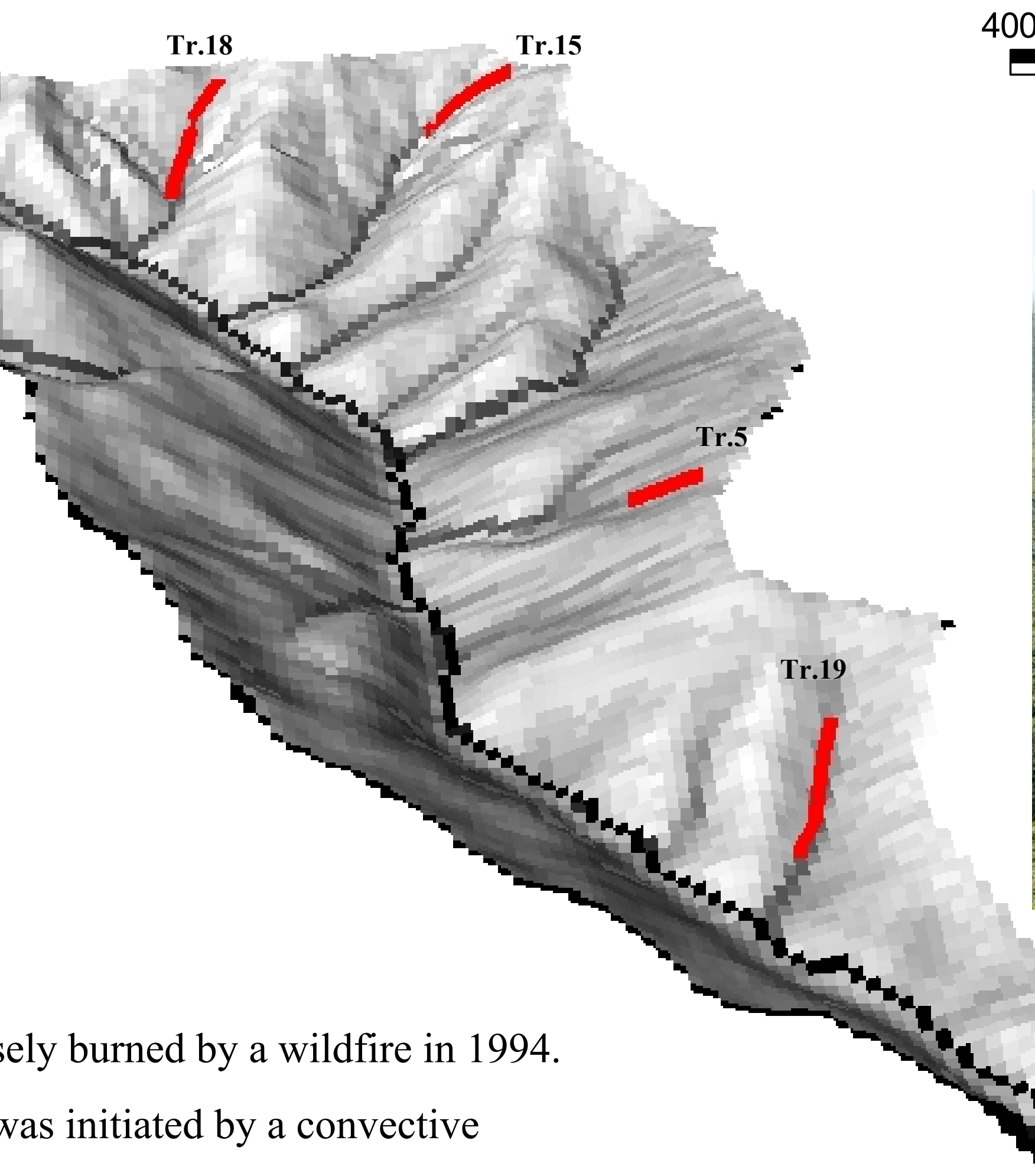
Abstract

We have conducted surveys of the gullies that developed in a small steep watershed in the Idaho Batholith after a severe wildfire followed by intense precipitation. We measured gully extent and cross sections and used these to estimate the volumes of sediment loss due to gully formation. These volume estimates are assumed to provide an estimate of sediment transport capacity at each survey cross section from the single gully forming thunderstorm. Sediment transport models commonly relate transport capacity to overland flow shear stress, which is related to runoff rates, slope and drainage area. We have estimated the runoff rates and duration associated with the gully forming event and in this paper used the sediment volume measurements to calibrate a general physically based sediment transport equation in this steep high shear stress environment. We find that a shear stress exponent of 3 which corresponds to drainage area and slope exponents of 2.1 and 2.25 match our data. This shear stress exponent of 3 is approximately two times higher than the exponents used for sediment transport in alluvial rivers, but in the range of shear stress exponents observed in flume experiments on steep slopes. In this poster we also coupled the calibrated sediment transport equation with the probabilistic approach for channel initiation (PCI) Istanbulluoglu et al. [2001] to show its use to predict expected sediment transport capacity over the terrain and sediment delivery to streams. Our results, although somewhat preliminary due to the uncertainty associated with the sediment volume estimates, suggest that for steep hillslopes such as those in our study area, a greater nonlinearity in the sediment transport function exist than that assumed in existing hillslope erosion models.

Study Site

The study watershed is Trapper Creek located on the North Fork of the Boise River in the Idaho Batholith.





Gullying;

- Trapper was intensely burned by a wildfire in 1994.
- Extreme gullying was initiated by a convective summer storm in 1995. Gully incisions started close to the ridge tops. On the average gullies were 2-3m deep and 3-4m wide.
- Our study of gullies focused on the west part of the watershed where the geology was relatively homogeneous.

Field Observations;

- Upslope extent of gully incisions.
- Volume of eroded material from gully cross-sections in 20-30 m intervals starting from the channel heads.
- Local slope at each cross-section.
- Sediment size.

Geology/Climate;

- Granitic bedrock.
- Mostly forested
- Extremely erodible coarse textured soils.
- Steep gradients often exceed 60%.
- Narrow and V shaped valleys.
- Episodic hollow evacuation.
- Localized high intensity thunderstorms during the summer and widespread storms often conjunction with snowmelt at other times.

Theoretical Analysis

Past Work on Sediment Transport In Rivers and Flumes

A general dimensionless sediment transport equation;

Many bedload sediment transport equations can be written in a dimensionless form;

$$q_{s*} = \beta \tau_*^{p_2} \qquad (1)$$

Where,

$$q_{s*} = \frac{q_s}{\sqrt{g(s-1)d^3}} \qquad (1a)$$

$$\beta = \kappa\left(1 - \frac{\tau_{*c}}{\tau_*}\right)^{p_3} \qquad (1b)$$

$$\tau_* = \frac{\tau/(\rho_w g)}{(s-1)d} \qquad (1c)$$

q_{s*} :dimensionless bedload sediment transport rate.
q_s :bedload sediment transport rate.
τ_* :dimensionless bed shear stress.
τ :bed shear stress ($\tau = \rho_w gRS$).
d : dominant sediment size.
β : dimensionless bedload rate parameter.
κ, p_2, p_3 : calibration parameters.
g : gravity of acceleration.
s : ratio of sediment to water density.
ρ_w: water density.
R: hydraulic radius.
S: slope.
Bed form resistance is neglected and often p_2=p_3.

Yalin [1977] showed that κ would be 17 at high values of τ_*. Many **κ values** were reported in the range of **4-40** in different equations [*Yalin*, 1977; *Simon and Senturk*; 1977]. Bedload equations for **rivers** often use **p_2=1.5** [*Yalin*, 1977], **$p_2 \cong$2.5** for sediment transport on **steep slopes** [*Govers*, 1992; *Rickenmann*, 1991.

Adaptation of the Sediment Transport Model to Incising Gullies

Physical modeling of sediment transport in incising gullies requires;

- Adoption of the dimensionless sediment transport equation for natural terrain.
- Calibration of κ, p_2 and p_3 using field data for gully sediment transport.

Adaptation of the dimensionless sediment transport equation to incising gullies

Flow rate and flow hydraulic characteristics along gullies are described in terms of contributing area **A** and slope **S**.

Discharge at a point on the gully network is assumed proportional to A,

$$Q = rA \qquad (2)$$

where r is runoff rate. Hydraulic radius is described as a function of flow cross-sectional area, A_f and a shape constant, C assuming top width to depth ratio of the flow is always constant (uniform enlargement of the flow cross-sectional area) [*Foster et al , 1984*; *Moore and Burch*, 1986],

$$R = CA_f^{0.5} \qquad (3)$$

Here, A_f=Q/V, and can be written proportional to A and S, using Manning's equation for V by implementing at-a-station hydraulic roughness, ***$n=k_nQ^{-m_n}$*** [*Knighton*, 1998] where k_n and m_n are empirical parameters. We wrote flow cross-sectional area, hydraulic radius, effective shear stress (shear stress acting on grains) and flow width in terms of A and S in a general form as,

$$\Psi = \chi_\Psi (rA)^{m_\Psi} S^{n_\Psi} \qquad (4)$$

Table 1. Physical parameters of the hydraulic variables in the form, $\Psi = \chi_\Psi Q^{m_\Psi} S^{n_\Psi}$.

Ψ	χ_Ψ	m_Ψ	n_Ψ
Flow cross-sectional area, A_f	$k_n^{0.375} C^{-0.5}$	0.75(1-m_n)	-0.375
Hydraulic radius , R	$k_n^{0.375} C^{0.75}$	0.375(1-m_n)	-0.1875
Flow width, W_f	$k_s k_n^{0.375} C^{-0.25}$	0.375(1-m_n)	-0.1875
Effective shear stress, τ_f	$\rho g C^{0.75} k_n^{-1.13} n_{gc}^{1.5}$	0.375+1.13m_n	0.8125

The flow width is obtained from the flow cross-sectional area by assuming a specific cross-section geometry. The parameter $k_s$is obtained from the cross-section geometry and is $z_1/(z_1-z_2)$ for trapezoidal channels, $2z_2^{0.5}$ for triangular channels and $(1.5z_1)^{0.5}$ for parabolic channels, where z_1 is the width/depth ratio and z_2 the side slope. The effective shear stress is obtained by using the grain roughness n_{gc} in Manning's equation to obtain an effective grain hydraulic radius R_{gc}. The effective shear stress is assumed to be the fraction R_{gc}/R of the total shear stress [*Laursen*, 1958; *Tiscareno-Lopez et al.*, 1994].

Total sediment transport capacity of the flow is the flow width times the unit sediment transport rate.

$$Q_s = W_f q_s \qquad (5)$$

This is obtained by substituting the effective stress in the form of (4) in Table 1 into the dimensionless sediment transport capacity equations (1) and solving (1) for q_s and substituting both the expression obtained for q_s and flow width in Table 1 into (5),

$$Q_s = [\kappa \chi_c^{-1} \chi_W \chi_\tau^{p_2} d^{1.5-p_2} (1-\tau_c \chi_\tau^{-1} r^{-m_\tau} A^{-m_\tau} S^{-n_\tau})^{p_2} r^M] A^M S^N \qquad (6)$$

where, $\chi_c = \rho^{p_2}(g(s-1))^{p_2-0.5}$ $\quad N = p_2 n_\tau + n_W \quad M = p_2 m_\tau + m_W$

When $\tau_c = 0$, equation (6) predicts that,

$$Q_s \propto A^M S^N \qquad (7)$$

This equation expresses sediment transport in terms of topographic variables.

Procedure for calibrating the sediment transport equation for incising gullies

Here we developed a procedure to obtain the required calibration parameters κ, and p_2 from field observations. We assume that once a gully is incised, the sediment transport rate is at its transport capacity for the duration of the gullying event. Based on this assumption, the average steady-state unit sediment discharge of a point in the gully is the total volume of sediment passing that point V_s divided by the total erosion duration T, and flow width W_f, which is written in a dimensionless form as,

$$q_{s*} = \frac{V_s}{T \chi_W r^{m_W} A^{m_w} S^{n_W} \sqrt{g(s-1)d^3}} \qquad (8)$$

We use the effective shear stress equation from Table 1 and substitute into Equation (1c) to write the dimensionless shear stress as,

$$\tau_* = \frac{\chi_\tau r^{m_\tau} A^{m_\tau} S^{n_\tau}}{\rho_w g(s-1)d} \qquad (9)$$

Now plotting the q_{s*} obtained from observed V_s, A and S versus $\tau_*(1-\tau_{*_c}/\tau_*)$ we may obtain the empirical parameters κ and p_2 in equation (1) by fitting a power function to the data. Here p_3 assumed equal to p_2. Note that $\tau_*^{'} = \tau_*(1-\tau_{*_c}/\tau_*)$ in the remainder of the poster.

Field Data

Gully No	Drainage Area (m^2)	Local Slope (m/m)	Total Gully Erosion (m^3)	Gully No	Drainage Area (m^2)	Local Slope (m/m)	Total Gully Erosion (m^3)
Tr.05	5460	0.36	2.4	Tr15	42040	0.34	240.0
	6690	0.35	21.3		43900	0.39	637.5
	9000	0.55	45.3		45880	0.45	726.0
	9600	0.55	83.2		50680	0.55	954.0
	12000	0.55	131.0		55780	0.50	1184.0
	12900	0.55	146.1		59950	0.49	1250.5
	16500	0.28	187.3		71800	0.41	1379.5
	22500	0.55	249.4		75940	0.40	1677.0
	24000	0.55	269.0		71260	0.49	2117.0
	27600	0.53	282.8		108760	0.32	3497.0
	30000	0.50	304.7		114640	0.40	4547.0
	33600	0.35	334.4	Tr.19	18000	0.45	16.5
	34440	0.33	395.3		38610	0.48	33.5
Tr.18	6300	0.51	0.36		46800	0.46	67.0
	8700	0.47	0.44		54000	0.45	116.5
	9000	0.61	0.89		60000	0.4	144.0
	15660	0.60	2.79		74550	0.4	171.5
	19020	0.64	12.29		87000	0.35	177.6
	22500	0.65	23.70		90000	0.35	207.2
	24000	0.50	34.07		94260	0.45	248.2
	26190	0.50	38.87		120000	0.32	273.2
	42000	0.60	58.87		125790	0.31	299.7
	45300	0.40	95.77		150000	0.18	418.4
	61620	0.40	178.27		156480	0.19	572.4
	69900	0.44	358.27		166740	0.2	604.2
	74880	0.43	478.27		188070	0.18	621.4
	81000	0.35	652.27		198750	0.17	628.9
	100200	0.29	936.27		210000	0.16	680.3
	102000	0.29	1126.27		216480	0.2	708.5
	107130	0.30	1696.27		222000	0.18	827.5
	115860	0.37	2576.27		228000	0.23	887.0
	135000	0.30	3176.27				
	158430	0.28	3921.27				
	210000	0.20	4946.27				
	253680	0.13	6096.27				
	268890	0.24	7176.27				
	276000	0.24	8046.27				

See watershed map under the study site section for locations of the four gullies listed above.

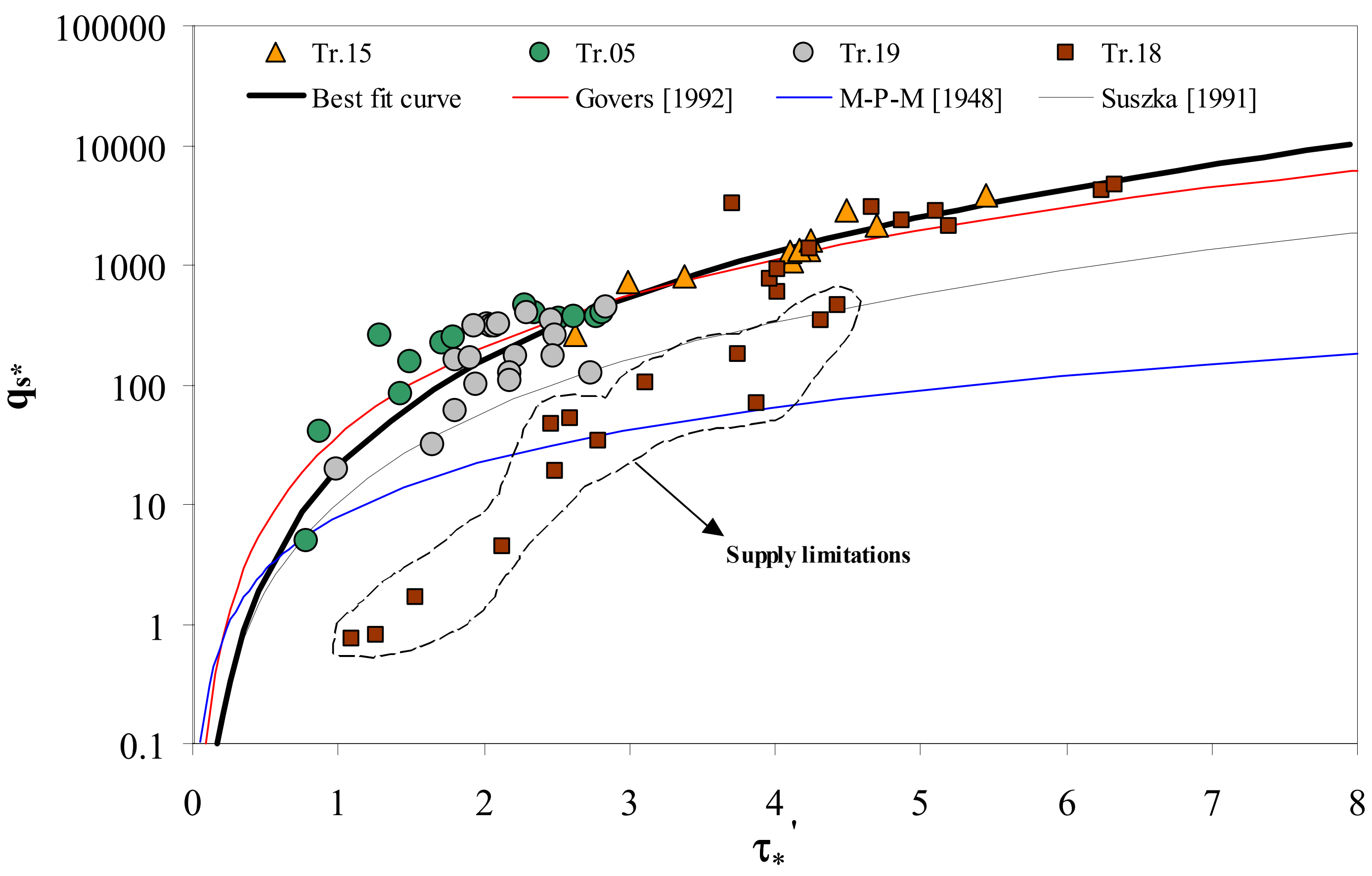
Parameter Inputs

Parameter	Value	Source
Median size of the eroded sediment	3 mm	Field observations
Manning's roughness coefficient for grains	0.025	Arcement and Schneider [1984]
Dimensionless critical shear stress	0.045	Suszka [1991]
At-a station hydraulic roughness exponent	0.2	Knighton [1998]
At-a station hydraulic roughness constant	0.045 for Tr. (5;15;18) 0.08 for Tr. 19	Field observations of the roughness elements and comparison with Arcement and Schneider [1984]
Runoff rate	30 mm/h	hypothesized based on the rainfall rate of ~50-70 mm/h on partially water-repellent soils
Sediment transport duration	0.5 h	hypothesized based on the information provided by some forest workers exposed to the event

These parameters are inserted into Equation (1) to obtain the sediment transport model parameters

Results

Calibration of the dimensionless sediment transport equation for incising gullies



- Relationship between q_{s*} as a function of $\tau_*^{'}$ is obtained using the field observations. The fitted relationship in the form of equation (1) has κ=20, p_2=3.

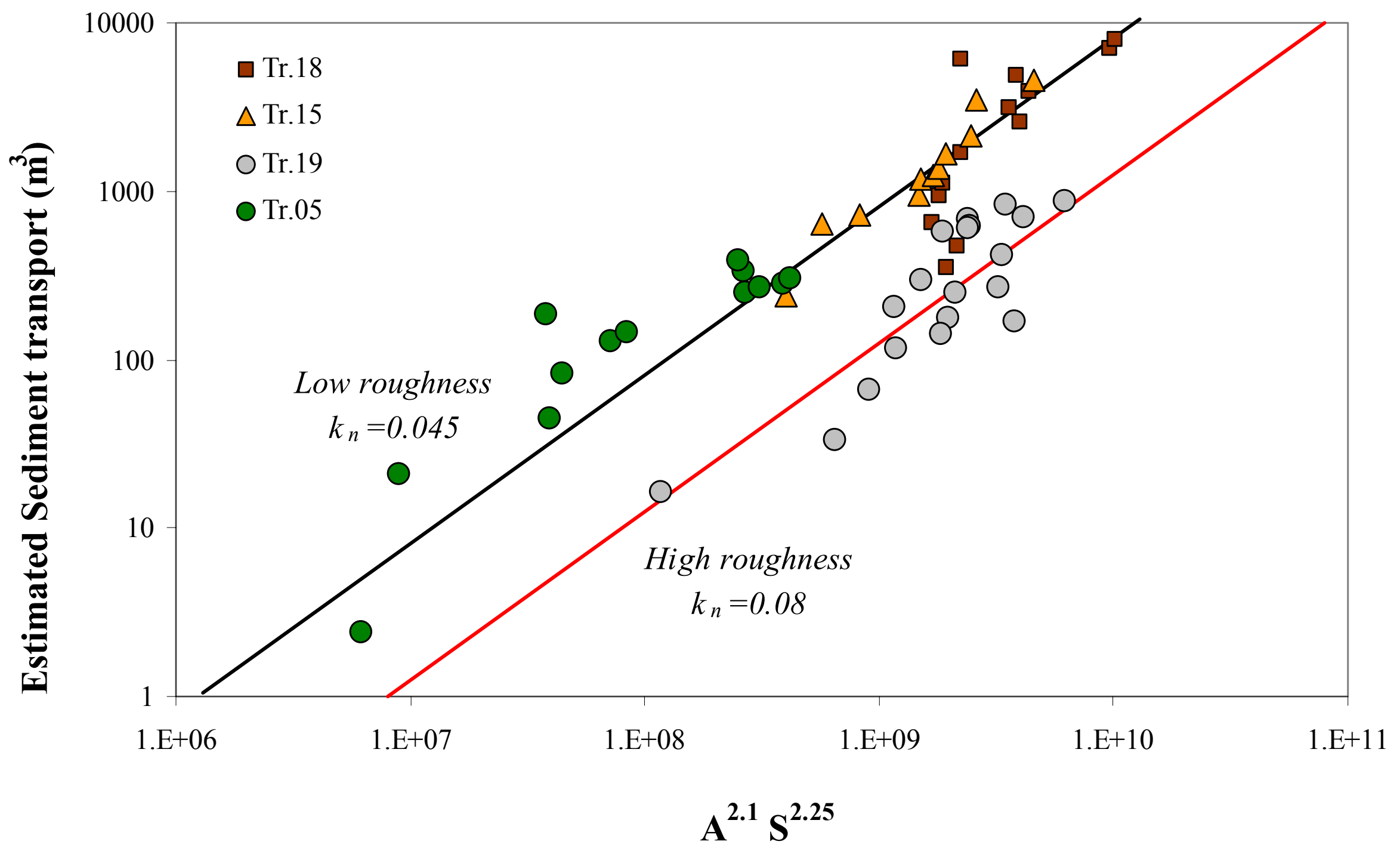
- Dashed lines highlight the sediment supply conditions in Tr.18, where sediment transport was initially supply limited due to discontinuities in the gully. Sediment transport rate reached its capacity following subsequent gully side wall collapses downslope.

The figure also compares the dimensionless forms of several sediment transport equations against the field data. Parameters for the equations are;

-Meyer-Peter and Muller [1948]; κ=8, p_2=1.5 (Reported for alluvial rivers).

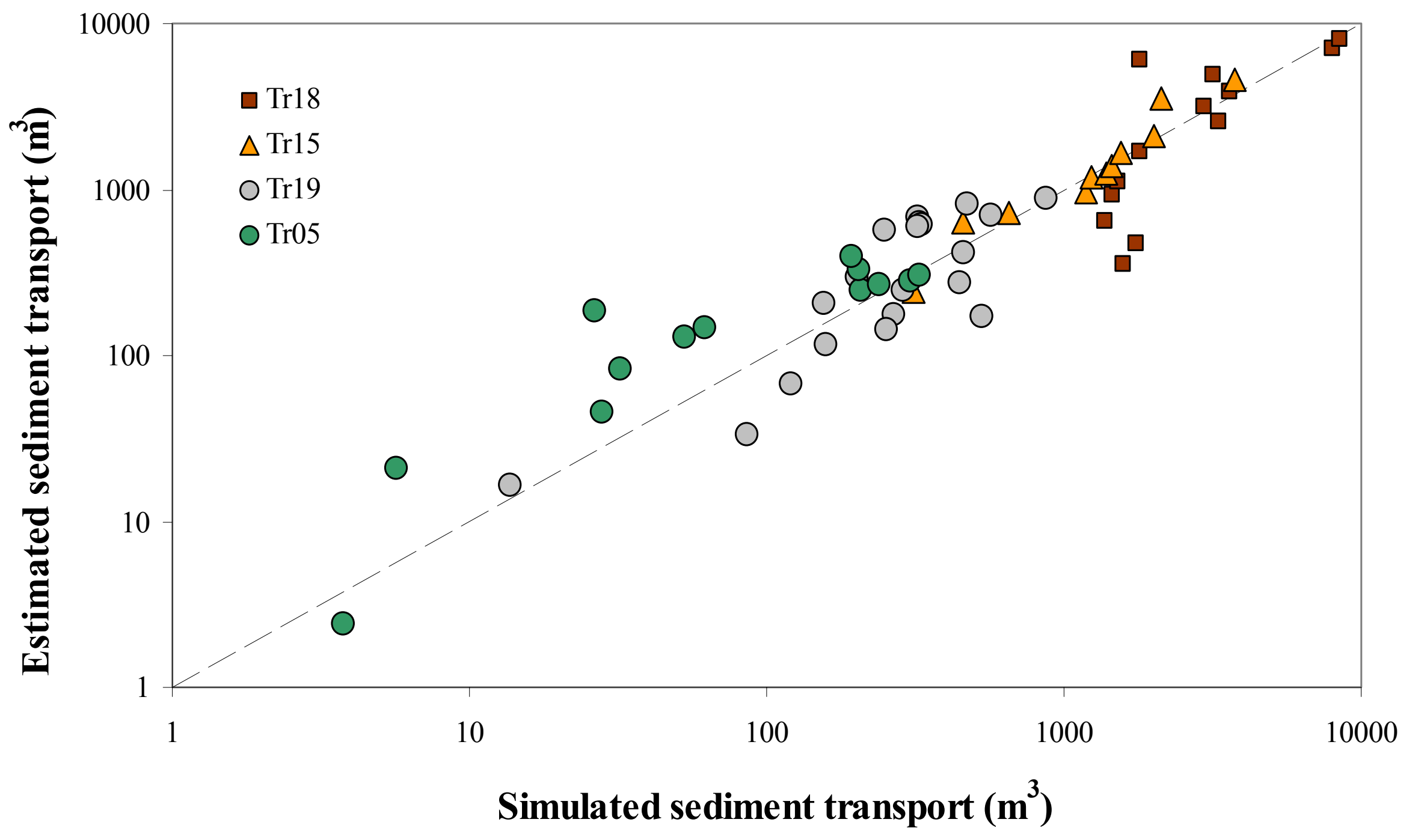
-Suszka [1991]; κ=10.4, p_2=2.5 (Reported for sediment transport under high shear stresses).

-Govers [1992]; κ=34.7(s-1)$^{1.957}$d$^{0.146}$, p_2=2.5 (Reported for overland flow on steep slopes. This equation is non-dimensionalized in the form of (1) for the analysis.)



Estimated sediment transport in the field reveals strong linear relationships with $A^M S^N$ at surveyed gully segments. The derived exponents are M=2.1 and N=2.25 (based on calibrated p_2=3)

The lines plot equation (6) for relatively low (Tr.5;15;18) and high (Tr.19) roughness conditions observed in the gullies. A parabolic cross section that has $k_s=(1.5z_1)^{0.5}$ with z_1=3, was assumed.

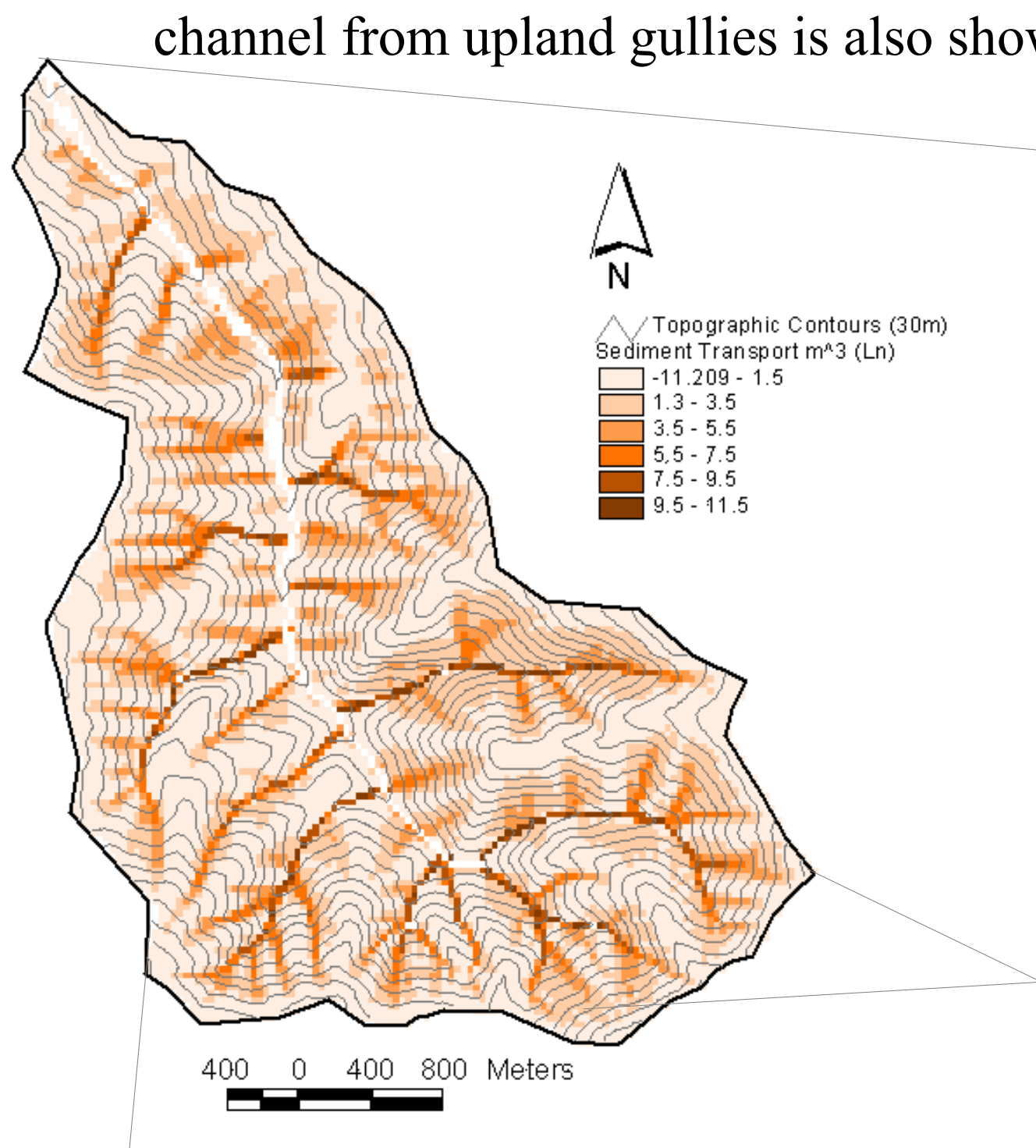


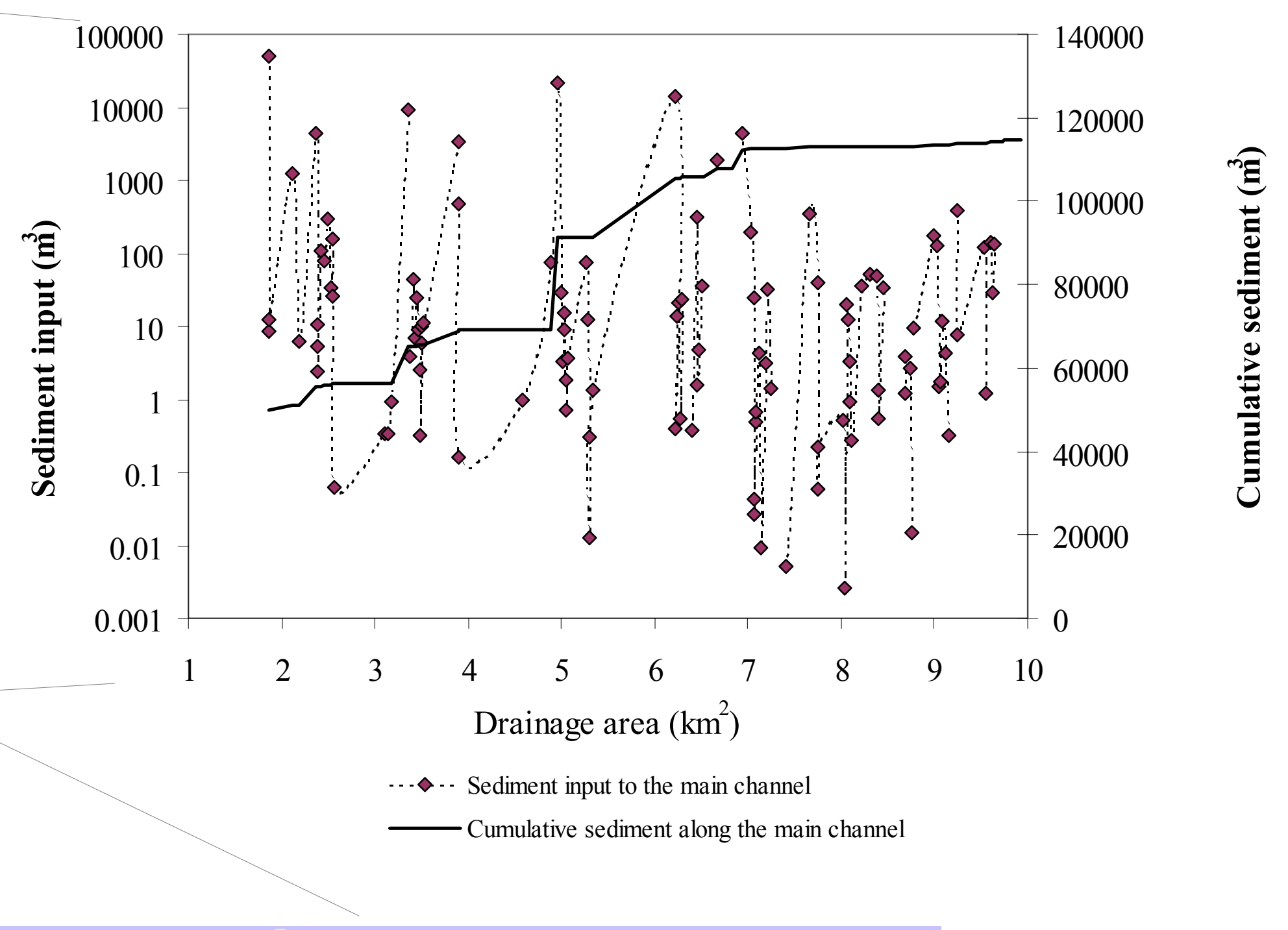
This figure plots the total sediment transport volumes calculated from equation (6) against field estimates of sediment transport.

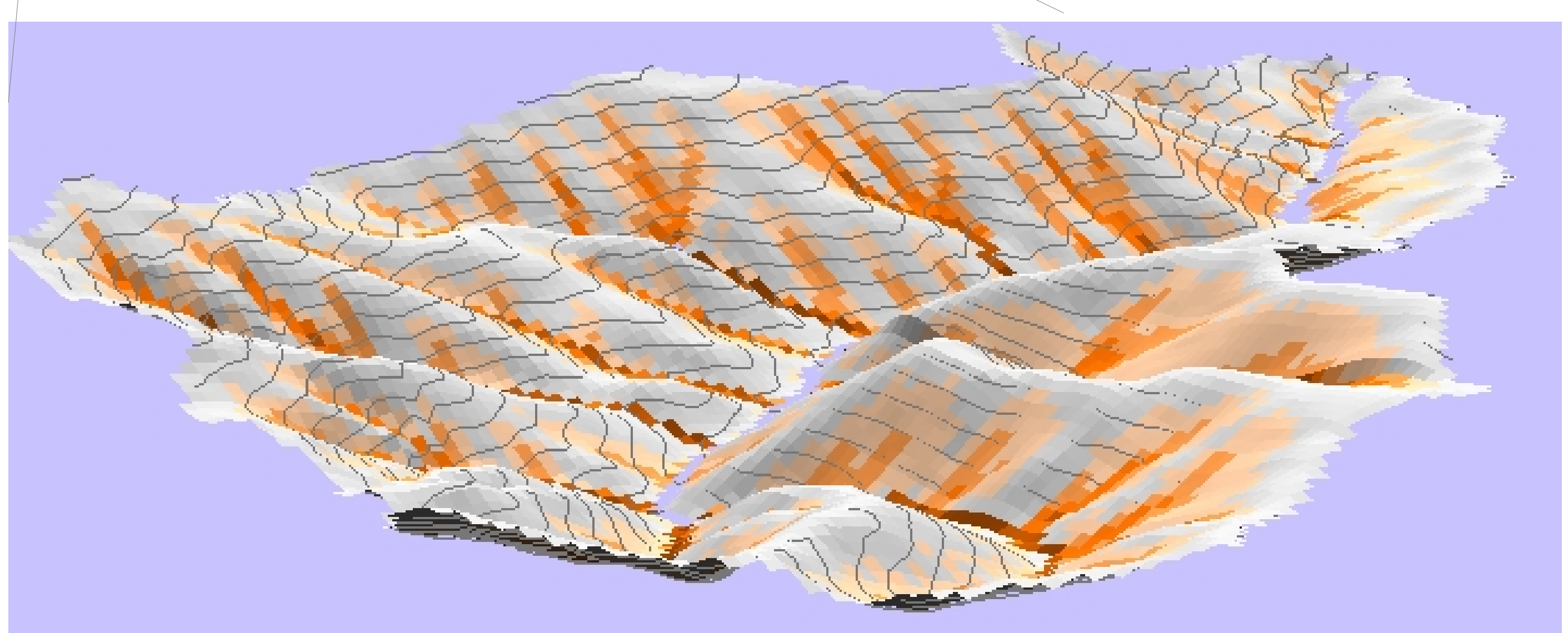
For the combined data of Tr.05, Tr.15 and Tr.18 both R^2 and Nash-Sutchlift error measure (NS) are 0.81. For the Tr.19 data R^2=0.5 and NS=0.44. Tr.19.

Modeling sediment transport on the watershed scale

Equation (6) is used to map gully sediment transport capacity over the terrain. Gully initiation is represented using a probabilistic channel initiation (PCI) approach [Istanbulluoglu et al., 2001]. The map here shows expected sediment transport calculated as the product of sediment transport capacity and PCI. Expected sediment input along the main channel from upland gullies is also shown.







Expected sediment transport capacity over the terrain

Conclusions

- **Sediment transport in gullies on steep topography is found to be a nonlinear function of shear stress with an exponent of 3. This exponent is two times higher than the exponents used for sediment transport in alluvial rivers but consistent with steep flume experiments for shallow flows [Govers, 1992] .**
- **A shear stress exponent of 3 is required to best fit the observed contributing area, local slope, and erosion field data regardless of the other input parameters used.**
- **A shear stress exponent of 3 theoretically corresponds to drainage area and slope exponents of 2.1 and 2.25 in the model. The tight relationship between the field estimates of sediment transport and $A^{2.1}S^{2.25}$ of measurement locations shows the importance of topography on sediment transport. The lack of scatter in the plots may suggest that possible spatial variations in the other model parameters along gullies do not significantly effect the transport rates.**
- **For the case of Tr.05, Tr.15 and Tr.18, 80% of the spatial variability of the sediment transport rates can be represented by the model whereas only 44% of the variability of the sediment transport rates in Tr 19 is explained. The reason for a significantly lower model performance in Tr.19 is we believe due to local non-transportable obstructions inside the gully which might violate the assumption of constant model parameters in the model. These obstructions reduce the sediment transport rates as well.**
- **Hillslope erosion models often use sediment transport equations developed for alluvial rivers with exponent 1.5. Here we suggest that there is a greater non-linearity in the sediment transport function than assumed in these existing models.**

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